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WEATHER FORECASTING IN TROPICAL REGIONS

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WEATHER FORECASTING IN TROPICAL REGIONS

This memoir was prepared for the Meteorological Research Committee in May 1946

PART I—TROPICAL FORECASTING IN GENERAL

§ 1—INTRODUCTORY

The tropical regions occupy nearly one half of the earth's surface and their economic importance, though not comparable with that of some areas of temperate latitudes, is very great, and is likely to increase with progressive development and settlement, and the education of the native peoples. The more productive areas not only support large populations, but also supply the populous industrial regions of the world with many foodstuffs and raw materials; while the ocean areas and desert or semi-desert areas, though largely unproductive and uninhabited, are traversed by important sea and air routes. In general, therefore, meteorology in the tropics is important in connexion with agriculture and communications, though, in contrast to temperate regions, there is less emphasis on the latter owing to the lower density and frequency of traffic. Because of this there has been a tendency in the past for meteorological services in the tropics to incline towards the climatological rather than the synoptic aspects of meteorology. The war, however, has led to considerable developments of the synoptic services to meet the needs of operational flying, and the necessity for these services, though now diminished, is likely to continue in view of the probable growth of civil aviation. Apart from agricultural and aviation requirements [illegible] is also made of weather forecasts by shipping interests and the general public.

§ 2—AVIATION FORECASTS

Though over mountainous areas and some coastal regions heavy cloud and rainfall occur frequently, it may be said broadly that weather conditions adverse for flying are rare in the tropics in comparison, say, with north-west Europe. Both over sea and land the predominating conditions are fair or fine, with well broken cloud at a safe height, good visibility and light or moderate winds. The occasions on which the meteorological service can be the means of saving lives or aircraft by advising cancellation or diversion of flights are comparatively few. When adverse conditions do arise, however, they are attended by special hazards. Diversionary landing grounds are limited in number and often poorly equipped, and for some transoceanic flights, using isolated islands as stopping places, they are non-existent. Many routes involve long flights over imperfectly charted inhospitable country, lacking well defined landmarks, where forced landings result in hardship and suffering and often danger to life. Over mountain areas, it is extremely risky for a pilot flying above cloud to come down to check his position; with modern radio aids to navigation this should not in general be necessary, but in the tropics the worst weather is usually associated with violent thundery conditions which make radio communication difficult if not impossible; under these conditions accurate forecasts are invaluable.

The main hazards to aircraft in the tropics are thunderstorms with their dangerously turbulent clouds, the obscuring of hills and landing grounds by low cloud or fog, duststorms in arid areas, and, over the sea, occasional squalls and tropical revolving storms. As has been said above, these adverse conditions are comparatively infrequent and limited in extent, but when they do occur they are usually violent and intense, and are all the more dangerous when they are encountered suddenly by pilots who are accustomed to flying in mainly fair weather.

In east Africa, the tropical region known to the present writer, low cloud over and around the destination airfield arises principally from widespread afternoon thunderstorms, and makes landing hazardous at a particular airfield on perhaps five days a year. It is important that warnings of these conditions be sent out in good time to allow incoming flights to be cancelled or diverted. The alternative landing grounds are usually at considerable distances, say one to three hundred miles, and the bad conditions rarely affect more than one of them simultaneously; long-distance aircraft usually carry enough petrol to reach them, so that the meteorological advice required is usually with a view to diversion rather than cancellation of these flights. At night the thunderstorms often spread out into large areas of light rain with overcast conditions, though, except in hilly areas, the cloud base is not, as a rule, dangerously low.

Early morning fog and low stratus occur in many tropical areas, and though they have not greatly affected flying in the past, except to cause delays in starting, with the development of night flying and dawn landings they come more into the picture. Hitherto in east Africa little success has been achieved in forecasting these conditions from the data available on the previous evening. As a rule they are present only for an hour or so following sunrise, but in particular orographic situations, and perhaps also in association with frontal zones, they may occasionally, as on the high ground around Nairobi, persist until the afternoon. They are, however, more localised than the thick cumulonimbus associated with widespread afternoon thunderstorms, and an aircraft can, as a rule, climb above them.

Convective processes in the tropics are very intense, and it is hazardous for an aircraft to fly into a cumulonimbus cloud at any time; by day these clouds can usually be avoided, but not always by night; therefore forecasts for night flying should include accurate information regarding them, particularly over land areas, where a pilot cannot always fly below them.

In certain areas tropical revolving storms give rise to very dangerous conditions to aircraft in flight, and even at great distances may cause swells which prohibit the take-off and landing of flying boats. Really severe cyclones are infrequent, though when they do occur their effects are catastrophic. Moderate cyclones are fairly frequent and give rise to stormy weather about as violent and widespread as that associated with deep winter-time depressions near the British Isles. The forecasting of tropical storms is therefore of the greatest importance for aircraft.

Upper winds are usually light or moderate, and are not liable to rapid variations either in space or time, so that they do not, as a rule, give rise to flying hazards. Improved forecasting of upper winds would, however, yield a useful return in that reserves of fuel now carried might be partly replaced by pay-load. Except for squalls which are usually short-lived, the surface winds at most airfields present no difficulties to pilots of aircraft, and in east Africa it has not been usual to include them in forecasts. They are important however at a few airfields, e.g. Nakuru, Kenya, where the topography gives rise to strong katabatic winds across the runways, for the hills which cause the katabatic flow do not permit aircraft to take off into the wind. One or two serious accidents to flying boats have occurred owing to landing down-wind, but these were not in any way due to meteorological advice.

Seasonal and diurnal effects are relatively much more marked in the tropics than in temperate latitudes, not because they are necessarily greater in themselves but because they are less masked by the irregular changes. It is therefore comparatively easy to forecast the general character of the weather for a long flight; it is, on the other hand, extremely difficult to forecast either the exact location of any adverse weather or the detailed sequence of weather at a particular airfield, and for the latter, the forecaster at the airfield, equipped with local knowledge, has a great advantage over his colleague at a central office. Some airfields become unserviceable after heavy rain, a condition of which a local forecaster may be able to give effective warning.

Synoptic charts supplied to pilots of long-distance aircraft are not of great value in the tropics. Rarely can they indicate satisfactorily the movement of bad weather associated with zones of convergence, for there is reason to believe that this movement is often discontinuous. In low latitudes little significance attaches to isobars, and well defined pressure systems with their characteristic weather are rare. The synoptic chart accompanying a forecast is therefore little else than a pictorial presentation of actual weather reports, and, on account of the pronounced diurnal variation of weather, a chart based on the latest reports prior to the flight may be misleading ; a chart based on conditions at stations 24 hours previous to the time at which the aircraft will be in their vicinity would be more informative.

The writer has no experience of the forecasting of duststorms, but they appear, as a rule, to be associated with fronts, which can be detected on the synoptic chart. They may render landing of aircraft impossible for some hours when they affect an airfield.

§ 3—FORECASTS FOR SHIPPING

Generally in the tropics forecasts are not so useful to shipping as in temperate latitudes on account of the infrequency of bad weather. Tropical revolving storms with their attendant gale or hurricane winds and high seas are the greatest danger, and it is one of the functions of the meteorological service to give timely warnings of these storms so that ships may avoid them. Squalls of limited extent and of short duration are fairly frequent near the equatorial convergence zone and other " fronts ", but in the present state of knowledge they cannot be forecast accurately ; as a rule they do not constitute a danger to shipping, being too transient and localised to raise high seas. Gales, apart from their association with tropical storms, are rare, and fogs are almost entirely restricted to certain well known areas, the western cold-water coasts of the continents, where they are an important climatic feature.

§ 4—FORECASTS FOR AGRICULTURE

In areas like east Africa where the rainfall is uncertain and the rate of evaporation very high [illegible] forecasts covering a few days or a week are extremely valuable, chiefly as guides to planting since the success of a crop is to a large extent dependent upon adequate moisture soon after planting. Seasonal forecasts would be of even greater value ; they would prevent the waste caused by planting seeds over large areas for which subsequent drought causes total crop failure. The forecast for 24 hours is not of great value for these operations.

Other weather elements such as temperature, frost, wind and sunshine have little effect on agriculture except in so far as they are features of the climate of a region, and are dependent to a large extent upon altitude ; synoptically the only element that really matters is rainfall.

§ 5—FORECASTS FOR THE GENERAL PUBLIC

The interest of the general public lies mainly in the question as to whether or not it is likely to rain, for in warm climates no other meteorological element, whose changes or incidence are irregular, gives rise to such inconvenience or discomfort. Heavy rain may render road travelling difficult if not impossible owing to mud and washaways, and in the rainy season at least, it is advisable to take meteorological advice before starting on a long journey. In certain areas tropical storms may cause loss of life and great damage, and their occurrence is therefore always a matter of concern to the whole community. Local and regional winds, for example the harmattan of west Africa and the monsoon, are important for the enervating or invigorating effects associated with their temperature and humidity.

§ 6—GENERAL TECHNIQUE OF TROPICAL FORECASTING

It is sometimes stated that forecasting in the tropics requires a different technique from that employed in temperate latitudes, but this is only partly true. The same basic dynamical and thermodynamical laws apply to the atmosphere, and the same influences are operative everywhere, though in differing degrees in different regions. Generally, the problems of forecasting are less amenable to mathematical or quasi-mathematical treatment in the tropics than in temperate latitudes; there is, for example, no simple approximate relation between the wind and the pressure distribution, and many of the manifestations of weather are instability phenomena and hence incapable of precise prediction. In temperate latitudes forecasting is based largely on the association between isobaric forms and the types of weather which accompany them; in the tropics well defined pressure systems are usually absent, and the isobars as a rule have no significance in relation to weather. Barometric tendencies, after correction for the regular oscillation, are often not much greater than the errors in the barometric readings themselves, especially when considerable corrections have to be applied on account of the reduction to a standard level, and their usefulness in forecasting is still uncertain, though they have been of assistance in detecting feeble troughs or wave-like disturbances moving over a group of stations near to sea level. In temperate latitudes convergence and divergence occur in association with certain features of the isobaric and isallobaric fields, and with fronts; the connexion is so close that the convergence or divergence is tacitly assumed in the appropriate situations. In the tropics on the other hand, convergence and divergence come into the picture, so to speak, in their own right, and must be estimated from the observed wind field; the synoptic chart must therefore be supplemented by upper wind charts. Finally, the absence of well defined pressure and frontal types with their more or less regular movement does not permit interpolation or extrapolation on the synoptic chart into areas from which observations are lacking, and in the tropics these are often very extensive.

§ 7—RELIABILITY OF OBSERVATIONS

The work of many of the native observers in tropical countries is surprisingly good, especially the pilot-balloon observing, but naturally it does not approach the high standard demanded at home. In particular, the observations of cloud type and height often seem very uncertain and the forecasters can have little confidence in them. It is evident that some observers fail to keep that close watch on the weather between synoptic hours which skilled and efficient reporting demands. Instrumental observations are on the whole quite good, better than those for which the exercise of judgment is necessary. But it does appear that, given the necessary initial training supplemented by fairly frequent visits to stations by meteorological officers, the native observer can attain a highly satisfactory standard. In the tropics, as elsewhere, the observations must be of the highest accuracy attainable.

§ 8—FREQUENCY OF REPORTS

In some tropical areas forecasting has in the past been severely handicapped owing to the synoptic reports not being made at frequent enough intervals. In the British Isles reports are now required hourly. Though, on account of the more regular diurnal variations, reports of this frequency are not necessary in the tropics, the absence of definite systems and progressions makes extrapolation much more difficult, and it is considered that the interval between synoptic reports should not be greater than 3 hours. The pronounced diurnal variation of weather may itself be adduced as an argument for this frequency, for reports made only twice daily may consistently miss important features of the weather and give no indication of the variations between the widely separated synoptic hours. For example, observations made daily at 0900 and 1500 (local time) only, miss the early morning fog or low cloud which occurs regularly at some stations at about sunrise but disperses before the time of the morning observation. Climatological summaries based on these reports are misleading.

§9—MAIN PROBLEMS OF TROPICAL FORECASTING

The main problems of tropical forecasting have been outlined by Hare[1*]; they are as follows:—

(*a*) The relation between the wind and the pressure distribution, including the estimation of convergence and divergence.

(*b*) Upper air and convectional phenomena.

(*c*) Frontal zones and discontinuities.

(*d*) Tropical revolving storms.

(*e*) Forecasting for agriculture.

The number of papers, published and unpublished, on tropical meteorology is very great, and it is clear from a perusal of many of them that the general problem has been attacked piecemeal. The most noteworthy contributions to the subject so far made have been by the Institute of Tropical Meteorology of the University of Chicago and by the Tropical Research Centre of the Royal Australian Air Force Meteorological Service, but most of the tropical meteorological services have made some contributions. The American writers deal chiefly with fronts in the tropics in a general way; the Australian writers on the other hand have restricted themselves in the main to the south-west Pacific and the region to the northward of Australia, but they have examined their own conclusions critically and produced supporting evidence in the form of observations and synoptic charts. The outstanding example of a work on tropical meteorology is, however, "Weather in the Indian Ocean, Part III, Aids to Forecasting"[2]. The writing of this publication and the analysis of the charts was carried out by Inst. Cdr. L. E. Pendal, R.N., of the Naval Meteorological Service while attached to the Meteorological Office. Though some of its conclusions are open to criticism this work presents a wealth of data sufficient for any reader so inclined to attempt re-analysis of the charts with which he disagrees.

PART II—RELATION BETWEEN WIND AND PRESSURE DISTRIBUTION

§10—INTRODUCTORY

The main difficulty in this problem arises from the weakness of the geostrophic force in low latitudes [illegible] the terms in the equations of motion which express the accelerations due to changing pressure distribution or departures from geostrophic flow become comparable in magnitude to the geostrophic term and must be taken account of in attempting to solve the equations. It is not to be expected therefore, that there will be even that approximate relation between the wind and the instantaneous isobars which holds in middle and high latitudes and is expressed by the geostrophic wind equation. Observation shows, in confirmation of this, that near the equator the isobars afford no guide to the surface and upper winds as regards either their speed or direction. Instances of the wind blowing more or less directly across the isobars from low to high pressure are by no means infrequent.

§11—PRACTICAL INVESTIGATIONS

A necessary first step is to determine from observations the extent to which the winds depart from geostrophic in low latitudes, and a comprehensive investigation on the lines of Gold's "Barometric gradient and wind force"[3] should be carried out for these regions. Preliminary investigations into the relation between mean pressure gradient and mean wind direction have been made by Jameson[4] for Ceylon, and more recently for an area in the South Atlantic. In the former it is difficult to allow for the effect of the high ground on the pressure distribution and the wind flow, but in the latter no such disturbing factor is involved. Jameson obtains the following mean values for the angle θ between the wind direction and the isobars.

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[*] The index numbers refer to the bibliography on p. 47.

Latitude (S.)	5°	10°	15°	20°
θ	45°	41°	32°	31°

This investigation should be extended to include the relation between barometric gradient and wind force, and should be carried out for other tropical areas. The south-west Pacific would appear to be a suitable region for this investigation, utilising the observations at the numerous island stations for which the pressures can be corrected to sea level without much error. Observations now available in this country for similar work are those from St. Brandon and neighbouring islands in the western Indian Ocean for parts of the years 1944 and 1945. St. Brandon (16° S., 59° E.) is a very small flat island with an excellent exposure for winds from all directions, and the gradient would be determined from the pressure there and at the other islands.

A brief examination of the east African synoptic charts and of the charts in Part III of " Weather in the Indian Ocean "[2] suggests that in the Mauritius area (in the trade-wind belt of the southern Indian Ocean) where the pressure distribution is almost steady, the departure of the surface wind from the geostrophic direction is very little greater than in temperate latitudes. The ratio of the surface wind speed to the geostrophic appears to be less than in temperate latitudes though not markedly so. These conclusions are in agreement with those obtained theoretically in § 16 below. The comparison is not strictly justifiable, for in temperate latitudes the average departure of the wind from the geostrophic is estimated from all observations, which are not restricted to cases of steady pressure distribution. Nearer the equator the investigation is more difficult, for the pressure gradients are usually very weak and the isobars are often based on stations so far apart that local gradients may exist and affect the wind and yet not appear in the isobaric pattern.

§ 12—EQUATIONS OF MOTION

The winds associated with a given distribution of pressure are theoretically determinate by means of the general equations of motion which relate the forces acting on the air on the one hand to the accelerations on the other. If it is assumed that the motion is horizontal, the equations are :—

$$\frac{du}{dt} - 2\omega v \sin\phi = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial z^2},$$

$$\frac{dv}{dt} + 2\omega u \sin\phi = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\frac{\partial^2 v}{\partial z^2};$$

x, y, z are the co-ordinates, the axes forming a right-handed system, x and y being horizontal and z vertical, positive upwards,

t is the time,

u, v, w are the components of velocity of the air,

ω is the angular velocity of the earth's rotation, $7 \cdot 29 \times 10^{-5}$ sec^{-1},

ϕ is the latitude, ϕ being reckoned positive in the northern and negative in the southern hemisphere,

ρ is the density of the air, and

ν is the kinematic viscosity of the air.

The terms on the right hand side express the forces acting on the air due to the pressure gradient and internal friction respectively. The terms du/dt, dv/dt express the ordinary accelerations, d/dt denoting differentiation following the motion. The terms in $\omega \sin \phi$ express the fact that air moving freely (and therefore unaccelerated relative to axes fixed in space) is accelerated relative to the system of axes rotating with the earth to which the motion is referred. The apparent deviating force which arises from the acceleration is the Coriolis, or geostrophic, force.

§ 13—GEOSTROPHIC WIND AND GRADIENT WIND

It is not possible to obtain a general solution of the equations of motion as they stand, but in temperate and high latitudes an approximate solution, which is justified by its agreement with observation, is obtained when certain simplifying assumptions are made. These are:—

(*a*) that the motion is unaccelerated,

(*b*) that the pressure distribution does not vary with time and that the gradient is uniform, the isobars then consisting of parallel straight lines, uniformly spaced,

(*c*) that friction can be neglected, the solution being taken as applying to the atmosphere above the level at which the frictional drag of the earth's surface is effective, and

(*d*) that the Coriolis force does not vary with latitude, or, alternatively, that the motion is confined to a small range of latitude over which sin ϕ may be treated as constant.

The equations of motion may then be reduced and combined into the well known geostrophic wind equation:—

$$V = \frac{1}{2\rho\omega \sin\phi} \cdot \frac{\partial p}{\partial n}$$

where V is the velocity of the air and $\partial p/\partial n$ the pressure gradient. The motion is parallel to the isobars and directed so as to keep the lower pressure to its left in the northern, and to its right in the southern hemisphere.

The solution can be extended to the case of steady motion in a circular path, leading to the gradient wind equation:—

$$\frac{1}{\rho}\frac{\partial p}{\partial n} = 2\omega V \sin\phi \pm \frac{V^2}{R} \cot\theta$$

where R is the radius of the earth and θ the angular radius of the small circle path. The upper sign applies to cyclonic and the lower to anticyclonic motion.

The immediate point of interest is how these solutions are modified in low latitudes. The geostrophic wind equation becomes invalid, for the pressure gradients are small and often indeterminate, the geostrophic force is also small on account of the factor sin ϕ, and the accelerations [illegible]ger be neglected in comparison with them. In steady motion the gradient wind equation [illegible]nue to be valid, the acceleration due to curvature of the path balancing that due to pressure gradient, and indeed such a balance is probably achieved in a tropical revolving storm. But in general the air movement in the tropics is not steady nor does it take place in small circles, so that the acceleration, apart from that due to curvature, is still important. Both equations are complicated by the fact that the most rapid variations of Coriolis force with latitude occur near the equator. Any solution of the equations of motion which is to be valid in the tropics must, therefore, take account of the acceleration and should also allow for the variation of the Coriolis force with latitude.

§ 14—GRIMES' SOLUTION FOR MOTION ACROSS THE EQUATOR

A solution which partly fulfils these requirements has been given by Grimes[5] and discussed and extended by Crossley.[6] This solution is subject to certain restrictions, the most important of which are:—

(*a*) that the motion is steady ($\partial u/\partial t = \partial v/\partial t = 0$),

(*b*) that the motion is independent of longitude ($\partial u/\partial x = \partial v/\partial x = 0$), and

(*c*) that the horizontal motion is non-divergent ($\partial u/\partial x + \partial v/\partial y = 0$).

The x and y axes are here assumed to be drawn towards east and north respectively.

Particular examples of stream-lines are then derived, taking as initial conditions specified values of the velocity and vorticity in a given latitude. These solutions, however, determine not only the stream-lines but also the pressure distribution, whereas the practical problem is to determine the motion from the specified pressure distribution and the initial velocities. The isobar and stream-line patterns thus obtained do indeed conform to the mean pressure and flow patterns of certain regions of the Indian and Pacific Oceans, but the above restrictions limit the solutions to mean conditions. The restriction to steady motion is contrary to experience in many parts of the tropics. Crossley[6] has shown, moreover, that on account of the semidiurnal variation of pressure the solutions apply only to mean motions and not to instantaneous motions. In his extensions to Grimes' work, Crossley has also removed restriction (*b*), but the solution thus obtained still determines both the motion and the pressure distribution.

The restrictions to non-divergent motion (implying that the vertical velocity w is zero under the assumption of constant density) not only leads to both the velocity and pressure being determined from the solution, but also evades the important question of convergence and divergence, which should be determined from the horizontal flow. The vertical velocity is usually omitted on the grounds that its inclusion renders the equations of motion intractable. It is thought however, that it may be possible to remove the restriction $\partial u/\partial x+\partial v/\partial y=0$ while still assuming that the terms involving w in the equations of motion are negligible, and Crossley[6] in fact goes some way towards justifying this assumption. If this is so, the problem becomes that of determining u and v from the given pressure distribution, and hence of calculating the horizontal divergence. The physical assumption is that the pressure field and the inertia of the moving air determine a field of flow which may involve convergence or divergence, but that for simplicity of treatment the resulting vertical velocities may be neglected in the dynamical equations. It is with this in view that the investigations outlined below have been pursued.

§ 15—A SIMPLIFIED TREATMENT OF ACCELERATED MOTION

It is possible to integrate the equations of motion taking account of a time-variation of pressure in a few special cases provided that the following simplifying assumptions are made:—

(*a*) that the variation of Coriolis force with latitude can be neglected;

(*b*) that the pressure field is delineated by straight isobars uniformly spaced, but the gradient and direction of the isobars may vary with time. At any instant the motion is everywhere the same so that the space derivatives in the acceleration terms do not appear in the equations of motion.

These assumptions impose limits on the validity of the solutions obtained, but they serve to indicate the nature of the motion in special cases and bring out some important differences between the motions appropriate to high and low latitudes. Assumption (*b*) means that the accelerations are given by $\partial u/\partial t$ and $\partial v/\partial t$ only, and the equations of motion are thereby reduced from partial to ordinary differential equations. The solutions may be taken as applying to the air so long as it remains under the influence of the same pressure field, say to motion within a 5-degree square. Then with velocities of up to, say, 50 Km./hr. the solutions would apply over a period of 12 hours. Their degree of approximation can, however, be found only by trial.

The equations of motion have been discussed on more general lines by Brunt and Douglas[7] and Sutcliffe[8], and some of the results given below are merely adaptations to specific pressure variations of the more general results given by these writers.

The simplified equations of motion have been used by a number of writers, notably by Gold[9] in relating the air movement to the diurnal variation of pressure, and by Sellick in connexion with periodic variations of pressure, in an unpublished paper shown to the present writer in 1944.

Let the pressure field be represented by

$$p = p_0 + A(t)y$$

where p_0 is constant and A is a function of t only. The isobars are then straight lines parallel, to the axis of x and uniformly spaced, but the spacing may vary with time.

Writing l for $2\omega \sin \phi$ and substituting for p we obtain the equations of motion in the form

$$\frac{du}{dt} = lv, \qquad \frac{dv}{dt} = -lu - \frac{A(t)}{\rho} \qquad \ldots\ldots(1)$$

in which, since there is only one independent variable t, ordinary and not partial differentials have been written.

(1) *General solution.*—The general solution of the equations (1) consists of two parts, the complementary function

$$\begin{aligned} u &= P\cos lt + Q\sin lt \\ v &= -P\sin lt + Q\cos lt \end{aligned}$$

in which P and Q are arbitrary constants, and the particular integral,

$$u = F(t), \qquad v = \frac{1}{l}\frac{dF(t)}{dt}.$$

where $F(t)$ is the particular integral of the differential equation

$$\frac{d^2\xi}{dt^2} + l^2\xi = -\frac{A(t)l}{\rho} \qquad \ldots\ldots(2)$$

The complete solution is therefore,

$$\left.\begin{aligned} u &= P\cos lt + Q\sin lt + F(t) \\ v &= -P\sin lt + Q\cos lt + \frac{F'(t)}{l} \end{aligned}\right\} \qquad \ldots\ldots(3)$$

where $F'(t)$ denotes $dF(t)/dt$.

The constants P, Q are determined from the initial conditions, say $u = u_0$, $v = v_0$ at $t = t_0$. We obtain

$$\left.\begin{aligned} P\cos lt_0 + Q\sin lt_0 &= u_0 - F(t_0), \\ -P\sin lt_0 + Q\cos lt_0 &= v_0 - \frac{F'(t_0)}{l}. \end{aligned}\right\} \qquad \ldots\ldots(4)$$

If $t_0 = 0$ these reduce to

$$\left.\begin{aligned} P &= u_0 - F(0), \\ Q &= v_0 - \frac{F'(0)}{l}. \end{aligned}\right\} \qquad \ldots\ldots(5)$$

(2) *Constant pressure gradient.*—$A(t)$ is constant and will be written simply as A.

$$F(t) = -A/\rho l \text{ and } F'(t) = 0.$$

Let u_g, v_g be the components of the geostrophic wind corresponding to the given pressure distribution.

Then $$u_g = -\frac{A}{\rho l} = F(t), \qquad v_g = 0 = F'(t).$$

Further let $u' = u_0 - u_g$ and $v' = v_0 - v_g = v_0$ so that u', v' are the ageostrophic components of wind at $t = 0$.

We obtain from (5)

$$P = u'. \qquad Q = v',$$

and the complete solution is therefore*

$$\left.\begin{aligned} u &= u' \cos lt + v' \sin lt + u_g, \\ v &= -u' \sin lt + v' \cos lt. \end{aligned}\right\} \qquad \ldots\ldots(6)$$

Superposed on the geostrophic motion is an oscillatory motion in which the amplitude of the velocity variation is $\sqrt{(u'^2+v'^2)}$ and the period is $2\pi/l$, i.e. one half pendulum day. The following table shows the approximate values of the period, in hours, for different latitudes.

Latitude	0°	5°	10°	15°	20°	25°	30°	60°
Period	∞	138	69	46	35	28	24	16

In temperate and high latitudes, where the departures from the geostrophic wind are on the average small, it is to be expected that the oscillatory motion will also be small. Brunt and Douglas[7] omit the oscillatory terms from their analysis, stating that no such oscillations have ever been detected. The " Meteorological Glossary " on the other hand states in the article on isallobaric wind that the harmonic oscillation would be difficult to detect because it would be damped out rather quickly by frictional effects, but important discrepancies, which have been found when attempts have been made to relate upper wind measurements to the pressure gradient and isallobaric gradient, are of an order quite compatible with the temporary existence of the initial oscillation.

The view is here put forward that under the conditions specified the ageostrophic wind is completely expressed by the oscillatory terms, which are therefore an essential part of the solution.

In the tropics the departures from geostrophic motion are known to be large at times, and in estimating the wind from the pressure distribution the oscillatory terms must be taken into account. Owing to the large value of the period $2\pi/l$ the periodicity of the motion corresponding to these terms will not at first be apparent, but there will merely be diminishing components $u'\cos lt$, $v'\cos lt$. When sufficient time has elapsed to show the periodic nature of the motion the pressure distribution will probably have changed and the initial conditions will no longer hold.

It is suggested that the equations (6) afford a means of relating the winds to the pressure distribution in the tropics. It is assumed that over a limited area the pressure gradient is uniform, and that it remains constant over a period of, say, 6 to 12 hours ; it is further assumed that the pressure gradient can be estimated from the synoptic chart. The geostrophic wind can therefore be evaluated.

Let the wind be measured at a particular instant taken as $t=0$. Then u' and v' can be obtained, and the numerical solution follows on substitution of the appropriate values in the equations (6).

It is to be noted that the calculated winds depend upon the pressure gradient, which is assumed constant, and the measured wind at a particular instant. The wind at some previous instant may be used to calculate the present wind, or the measured present wind may be used to calculate the wind at any future instant within the period for which the solution is valid. The calculation could, if required, be applied in extratropical as well as tropical latitudes.

The assumptions made in applying this solution are similar to those made in the case of the geostrophic wind equation, the principal one being that the pressure gradient is constant and uniform.

A trial should now be carried out to see how the results obtained by means of this solution compare with observation.

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* These equations have been given in a slightly different form by Hesselberg, see p. 46, Bibliography No. 10.

(3) *Solutions for particular time-variations of pressure.*—Exact solutions of the equations (1) can be obtained for any form of the function $A(t)$ which permits the particular integral of the equation (2) to be found. Among the forms which yield solutions are the polynomial, exponential and circular functions. Solutions will therefore be possible for pressure variations which ordinarily occur provided that these variations can be expressed in the above forms. Some examples are discussed below.

It is assumed that the pressure gradient changes smoothly from the constant magnitude B to the constant magnitude C without change of direction. The variation is most simply expressed by

$$p = p_0 + \left\{ \frac{B+C}{2} + \frac{C-B}{2} \tanh \beta t \right\} y \qquad \ldots\ldots(7)$$

where β is constant. At $t = -\infty$ this expression reduces to $p = p_0 + By$, from which, as t increases, it departs asymptotically, attaining its maximum rate of change at $t=0$, and as t increases further it asymptotically approaches the value $p = p_0 + Cy$. This variation is shown in Fig. 1, in which time varies along the horizontal axis and pressure gradient, G, along the vertical axis.

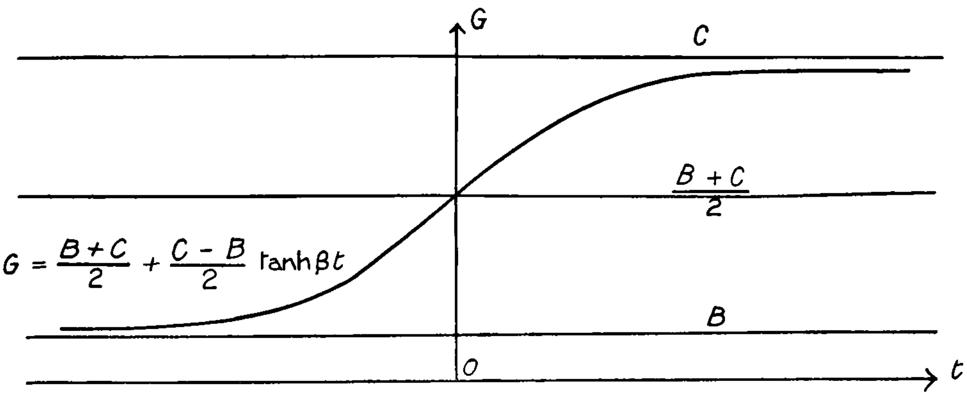

FIG. 1 - SMOOTHLY CHANGING PRESSURE GRADIENT

On account of the factor $\tanh \beta t$ the differential equations are not integrable, so that this example will not be considered any further.

A similar variation of pressure, for which the differential equations are integrable, is given by

$$\left.\begin{aligned} p &= p_0 + \left(B - \frac{B-C}{2} e^{\beta t}\right) y \quad \text{for } -\infty \leqslant t \leqslant 0, \\ p &= p_0 + \left(C + \frac{B-C}{2} e^{-\beta t}\right) y \quad \text{for } 0 \leqslant t \leqslant +\infty. \end{aligned}\right| \qquad \ldots\ldots(8)$$

The pressure is continuous at $t = 0$, and so also is $\partial p/\partial t$, but $\partial^2 p/\partial t^2$ and some of the higher derivatives are discontinuous.

The gradient is equal to B at $t = -\infty$ and C at $t = +\infty$. The rate of change of pressure is determined by β, being rapid if β is large and gradual if β is small. The curve of pressure gradient against time is similar to that in Fig. 1.

As the initial condition it is assumed that at $t = -\infty$ the wind is geostrophic, i.e.

$$u_0 = -B/\rho l, \qquad v_0 = 0 \quad \text{at } t = t_0 = -\infty.$$

For $t \leqslant 0$, applying (2) above, we obtain

$$F(t) = -\frac{B}{\rho l} + \frac{(B-C)l}{2\rho(l^2+\beta^2)} e^{\beta t}, \qquad F'(t) = \frac{\beta(B-C)l}{2\rho(l^2+\beta^2)} e^{\beta t},$$

and therefore, $F(t_0) = -B/\rho l, \qquad F'(t_0) = 0.$

Hence from (4),

$$P\cos lt_0 + Q\sin lt_0 = u_0 - F(t_0) = 0,$$

$$-P\sin lt_0 + Q\cos lt_0 = v_0 - \frac{F'(t_0)}{l} = 0,$$

so that $\qquad P = Q = 0.$

The oscillatory terms do not therefore appear in the motion for $t \leqslant 0$, which, from (3), is given by

$$\left.\begin{aligned} u &= -\frac{B}{\rho l} + \frac{(B-C)l}{2\rho(l^2+\beta^2)}e^{\beta t}, \\ v &= \frac{\beta(B-C)}{2\rho(l^2+\beta^2)}e^{\beta t}. \end{aligned}\right| \qquad \ldots\ldots(9)$$

For $t \geqslant 0$ we obtain similarly,

$$F(t) = -\frac{C}{\rho l} - \frac{(B-C)l}{2\rho(l^2+\beta^2)}e^{-\beta t}, \qquad F'(t) = \frac{\beta(B-C)l}{2\rho(l^2+\beta^2)}e^{-\beta t}.$$

The initial conditions are now obtained by putting $t = 0$ in (9), giving

$$u_0 = -\frac{B}{\rho l} + \frac{(B-C)l}{2\rho(l^2+\beta^2)}, \qquad v_0 = \frac{\beta(B-C)}{2\rho(l^2+\beta^2)} \quad \text{at } t = t_0 = 0.$$

Hence from (5),

$$P = \frac{(C-B)}{\rho}\,\frac{\beta^2}{l(l^2+\beta^2)}, \qquad Q = 0.$$

The solution for $t \geqslant 0$ is therefore, from (3),

$$\left.\begin{aligned} u &= \frac{C-B}{\rho}\,\frac{\beta^2}{l(l^2+\beta^2)}\cos lt - \frac{C}{\rho l} - \frac{(B-C)l}{2\rho(l^2+\beta^2)}e^{-\beta t}, \\ v &= \frac{B-C}{\rho}\,\frac{\beta^2}{l(l^2+\beta^2)}\sin lt + \frac{\beta(B-C)}{2\rho(l^2+\beta^2)}e^{-\beta t}. \end{aligned}\right| \qquad \ldots\ldots(10)$$

The solutions (9) and (10) exemplify certain features of the motion which are discussed from a more general point of view in the following paragraphs.

The amplitude of the oscillatory motion[10] is, from (10), proportional to the total change of pressure gradient. It also depends upon the relative values of l and β; if β is large in comparison with l the amplitude is large, and *vice versa* (near the equator) where l is small β must be very small if the oscillatory motion is to be negligible. It would be easy, though somewhat laborious, to calculate tables showing the amplitude for series of values of $(C - B)$, l and β.

The sine and cosine terms do not appear in the solution for $t \leqslant 0$, suggesting that if the motion is initially geostrophic and the pressure variation is smooth the oscillations will be absent. It appears that the oscillations arise as a result of the discontinuity at $t = 0$.

Suppose that at $t \leqslant 0$ we have a non-oscillatory solution given by

$$u = F_1(t), \qquad v = \frac{F_1'(t)}{l}, \qquad \ldots\ldots(11)$$

where $F_1(t)$ denotes the appropriate form of $F(t)$ given by (2). For $t \geqslant 0$ let the appropriate form of $F(t)$ be $F_2(t)$ so that the solution is

$$\left.\begin{aligned} u &= P\cos lt + Q\sin lt + F_2(t), \\ v &= -P\cos lt + Q\cos lt + \frac{F_2'(t)}{l}. \end{aligned}\right\} \qquad \ldots\ldots(12)$$

The initial conditions are obtained by putting $t = 0$ in (11), thus,

$$u_0 = F_1(0), \qquad v_0 = \frac{F_1'(0)}{l}.$$

Applying (5) to (12) we have, therefore,

$$P = F_1(0) - F_2(0), \qquad Q = \frac{F_1'(0)}{l} - \frac{F_2'(0)}{l}. \qquad \ldots\ldots(13)$$

At $t = 0$ there is a change in the function which expresses the variation of the pressure gradient with time. The equations (13) show that this change will not introduce oscillations provided that the particular integral $F(t)$ and its first derivative $dF(t)/dt$ are continuous at the change. This is a particular case of a general conclusion reached by Brunt and Douglas[7].

If the changes of pressure which occur in nature are smooth and represented by continuous functions—and there is no reason to suppose that in general they are not—the oscillatory motion is not present. The variation of pressure may, as in the first example on p. 13, involve non-integrable equations, but it may be presumed that the particular integral, though not calculable, is continuous. On this view therefore the oscillations arise only in the formal expressions for the motion because of the artificial discontinuity introduced into $F(t)$ or $dF(t)/dt$ in order to make the equations integrable.

On the other hand, the sudden changes of pressure gradient such as occur at fronts may perhaps be regarded as real discontinuities, and may therefore give rise to oscillations in the subsequent motion.

The geostrophic wind corresponding to the pressure distribution, (8), for $t \leqslant 0$ is

$$u_g = -\frac{B}{\rho l} + \frac{B - C}{2\rho l}\, e^{\beta t}, \qquad v_g = 0.$$

From (9) we obtain, writing $u' = u - u_g$, $v' = v - v_g$,

$$u' = \frac{C - B}{2\rho}\,\frac{\beta^2}{l\,(l^2 + \beta^2)}\, e^{\beta t}, \qquad v' = \frac{\beta\,(B - C)}{2\rho\,(l^2 + \beta^2)}\, e^{\beta t}. \qquad \ldots\ldots(14)$$

Similarly, for $t \geqslant 0$ we have

$$u_g = -\frac{C}{\rho l} - \frac{B - C}{2\rho l}\, e^{-\beta t}, \qquad v_g = 0.$$

Write

$$\bar{u} = u - \frac{C - B}{\rho}\,\frac{\beta^2}{l\,(l^2 + \beta^2)}\cos lt = -\frac{C}{\rho l} - \frac{(B - C)\,l}{2\rho\,(l^2 + \beta^2)}\, e^{-\beta t},$$

$$\bar{v} = v - \frac{B - C}{\rho}\,\frac{\beta^2}{l\,(l^2 + \beta^2)}\sin lt = \frac{\beta\,(B - C)}{2\rho\,(l^2 + \beta^2)}\, e^{-\beta t},$$

i.e. $\bar{u}$ and $\bar{v}$ are the component velocities given by (10) omitting the oscillatory terms. Then, if $u'' = \bar{u} - u_g$, $v'' = v - v_g$ we obtain from (10)

$$u'' = \frac{B - C}{2\rho}\,\frac{\beta^2}{l\,(l^2 + \beta^2)}\, e^{-\beta t}, \qquad v'' = \frac{\beta\,(B - C)}{2\rho\,(l^2 + \beta^2)}\, e^{-\beta t}. \qquad \ldots\ldots(15)$$

In (14) u' and v' are the components of ageostrophic wind; in (15) u'' and v'' are the non-oscillatory parts of the components of ageostrophic wind. They arise on account of the time variation of pressure, and hence may be called the components of the isallobaric wind. It is to be remarked that the amplitude of the oscillation given by (10) is no longer simply related to the components of geostrophic departure at the initial instant as in the case of the constant pressure gradient in § 15 (2).

From (14) we obtain $u'/v' = -\beta/l$ and from (15) $u''/v'' = \beta/l$, showing that in low latitudes, where l is small, the component along the isobars (and also along the isallobars) is large in comparison with the cross component. This appears to be incorrect when it is considered that the changing pressure gradient gives rise initially to the cross component of wind, and it is the action of the geostrophic force on this wind which produces the component along the isobars. The apparent anomaly arises from the fact that we are here considering not actual winds, but departures from geostrophic winds.

If in (9) we write $U = u - u_0$, $V = v - v_0$ where u_0 and v_0 are the components of wind at $t = t_0 = -\infty$, then U and V are the components of total wind change from t_0 to t.

We then have

$$U = \frac{(B - C)\,l}{2\rho\,(l^2 + \beta^2)}\,e^{\beta t}, \qquad V = \frac{(B - C)\,\beta}{2\rho\,(l^2 + \beta^2)}\,e^{\beta t},$$

giving $U/V = l/\beta$ and showing that in low latitudes the cross component of total wind change is the greater as would be expected. The isallobaric component u'' is large because the velocity u does not respond immediately to the changing pressure gradient, but is subject to a marked lag relative to the increasing geostrophic wind.

Solutions of the differential equations are readily obtainable for pressure variations of the types $p = p_0 + Ay\,\cos\beta t$ and $p = p_0 + Ay\,\sin\beta t$.

The details of the solutions will not be discussed here. If the initial motion is correctly adjusted the oscillatory terms will not appear in the solution. If $\beta = l$ we have the case of resonance, for which the solution gives infinitely large amplitudes to the forced oscillations. The oscillations would of course be damped by friction, but even so might still be large. It is not to be expected however that an applied oscillation of the appropriate period would be maintained long enough to give rise to marked resonance effects, nevertheless it might exist temporarily and give rise to surges of wind. This is a question which should be investigated.

Any arbitrary change of pressure could be made amenable to mathematical treatment by expressing it as a Fourier series.

A pressure field for which the gradient changes in direction as well as intensity can be represented in a form suitable for mathematical treatment by the expression

$$p = p_0 + Ae^{-\beta t}x + B\,(1 - e^{-\gamma t})\,y,$$

where A, B, β and γ are constants. The solution might be applied to curved isobars.

§ 16—EFFECT OF FRICTION

(1) *Surface friction.*—A number of writers have attacked this problem using the assumption that the motion is everywhere directly opposed by a force proportional to the velocity. This may be approximately true for the layer of air in contact with the earth's surface, but it neglects the internal frictional effects. Its justification is that it is the only plausible assumption which leads to tractable differential equations, and all the examples of § 15 can be extended to include friction of this character. The solutions are similar to those of § 15, so that the characteristics of the motion are preserved, but the velocities decay exponentially with time and phase differences are introduced into the oscillatory motions.

Taylor[11], using observations made by Dobson[12], has shown that the frictional resistance at the earth's surface is proportional to the square of the velocity. On this hypothesis the simplified equations for accelerated motion treated in § 15 are not generally soluble, but solutions can be obtained for the following special cases :

(*a*) steady motion under the influence of a constant uniform pressure gradient ;

(*b*) accelerated motion of air having a given initial velocity in a pressure field of zero gradient ;

(*c*) accelerated motion under a constant and uniform pressure gradient, neglecting the Coriolis force.

The oscillatory terms of period $2\pi/l$ appear in the solution for (*b*) but the amplitude varies as $1/t$.

This assumption however also neglects the internal friction and the results to which it leads are therefore of little practical interest.

(2) *Internal friction, case of steady motion.*—The terms $\nu\partial^2u/\partial z^2$ and $\nu\partial^2v/\partial z^2$ in the equations of motion of § 12 express the forces acting on an element of air due to internal friction. They are formally the same as the terms expressing the viscous forces in the standard hydrodynamical equations, but in turbulent motion it is the eddy viscosity and not the molecular viscosity which is effective. The ratio of the two coefficients of viscosity varies with the degree of turbulence of the atmosphere ; it is usually of the order 10^3 to 10^5.

Friction at the earth's surface also comes into play, exerting a drag, which, as stated in (1) above, is proportional to the square of the velocity. The mathematical expression of this fact is the boundary condition which is necessary in solving the equations of motion when the eddy viscosity terms are included. The equations were first solved for steady motion by Hesselberg and Sverdrup in 1915 and have been discussed by G. I. Taylor, Ekman and others. The following outline treatment follows that of Taylor[13] but with special consideration of the probable effects of friction in the tropics as compared with temperate latitudes.

The differential equations for horizontal motion under the influence of the pressure distribution $p = p_0 + Ay$, where A is constant, are

$$\frac{\partial u}{\partial t} = lv + \nu\frac{\partial^2 u}{\partial z^2}, \qquad \frac{\partial v}{\partial t} = -lu + \nu\frac{\partial^2 v}{\partial z^2} - \frac{A}{\rho}. \qquad \ldots\ldots(16)$$

where, as in § 15, the space derivatives in the acceleration terms have been ignored ; the symbols have the same meanings as in § 12.

The solution for steady motion $\partial u/\partial t = \partial v/\partial t = 0$, given by Taylor, is

$$u = V + e^{-pz}\{V_0 \cos(\theta - pz) - V \cos pz\},$$

$$v = e^{-pz}\{V_0 \sin(\theta - pz) + V \sin pz\},$$

where V is the speed of the geostrophic wind corresponding to the pressure gradient A, V_0 is the wind speed at the surface ($z = 0$), θ is the angle at which the surface wind cuts the isobars and p is equal to $\sqrt{(l/2\nu)}$.

By assuming that the frictional drag at the earth's surface acts in the direction exactly opposite to that of the wind, Taylor obtains

$$V_0 = V(\cos\theta - \sin\theta).$$

It follows that θ cannot, in steady motion, exceed 45°. The fact that values of θ in excess of 45° are frequently observed in the tropics may be taken as indicating that the motion is often unsteady.

If it be further assumed that the drag is equal to kV_0^2, where k is a constant for a given surface, we obtain the additional relation

$$\frac{\sin\theta}{1-\sin 2\theta}=\frac{kV}{\sqrt{(2l\nu)}}=\frac{\xi}{\sqrt{l}}\ \text{(say)}. \qquad \ldots\ldots(17)$$

This enables us to investigate the variation of θ, and hence of V_0/V, with latitude, for given conditions regarding wind and friction.

For moderate winds over Salisbury Plain (lat. 51°), Dobson[12] has found, approximately,

$$\theta=20^\circ, \qquad \frac{V_0}{V}=0\cdot 6.$$

From these figures, ξ in equation (17) can be evaluated, and using the value thus obtained we may derive the relation between θ and l. Solving graphically, we then obtain the following values of θ and V_0/V for various latitudes.

Latitude	0°	5°	10°	20°	30°	51° (Salisbury Plain)
θ	45°	28·5°	26°	23·5°	22°	20°
V_0/V	0	0·40	0·46	0·52	0·55	0·60

These results indicate that under the assumption of steady motion the modification, due to latitude, of the frictional effect is fairly small, except quite near the equator. It is certainly a good deal smaller than has hitherto been deduced on the simpler but inadequate assumptions that the motion is resisted by a frictional force proportional to the velocity or to the square of the velocity.

(3) *Special solution for accelerated motion, with friction.*—The general solution of the equations (16) with arbitrary initial values of u and v (which are now functions of z) is difficult. A solution has however been obtained for the case in which the air is initially at rest, the pressure gradient being constant and uniform. This solution is

$$\begin{aligned} u = V\,\{1-\sin\theta\, e^{-pz}\,[\cos(\theta-pz)+\sin(\theta-pz)]\}& \\ +\cos lt\sum_{n=1}^{\infty} e^{-\nu q_n^2 t}\,(C_n\cos q_n z+D_n\sin q_n z)& \\ +\sin lt\sum_{n=1}^{\infty} e^{-\nu q_n^2 t}\,(P_n\cos q_n z+Q_n\sin q_n z)& \qquad \ldots\ldots(18) \end{aligned}$$

with a similar expression for v. The symbols have the same meanings as in § 16 (2) with, in addition, $q_n = 2\pi n/h$, n being a positive integer and h the height of the layer considered (conveniently taken as the lowest level at which in steady motion the wind direction is geostrophic). By putting $t = 0$ the constants C_n etc., can be obtained in the following way :—

$$\begin{aligned} u_0 = 0 = V\,\{1-\sin\theta\, e^{-pz}\,[\cos(\theta-pz)+\sin(\theta-pz)]\}& \\ +\sum_{n=1}^{\infty}(C_n\cos q_n z+D_n\sin q_n z).& \qquad \ldots\ldots(19) \end{aligned}$$

The term in V is the solution for steady motion. C_n and D_n can be evaluated from (19) by the method of Fourier. From the expression for v the constants P_n, Q_n can be evaluated in a similar way ; the expressions for C_n etc. are very cumbersome.

Though equation (18) does not represent a motion which is likely to occur in practice it serves to indicate some features which may be regarded as characteristic of accelerated motion with friction. The negative exponential factors show that the steady state is approached asymptotically as the time increases ; they also indicate that the oscillatory motion dies out in the layer affected by friction. It is of interest to consider the rate of decay of the oscillations. Since they are represented by series of terms for which n takes integral values, we consider only the most slowly decaying term, that for which $n = 1$.

We have $q_1 = 2\pi/h$.

It has been shown by Taylor[11], that $h = (\theta + 3\pi/4)\ \sqrt{(2\nu/l)}$. Now θ and $1/\sqrt{l}$ each increase as the latitude decreases ; therefore q_1 decreases, the oscillations become more persistent and h increases, with decreasing latitude.

Take $\nu = 5 \times 10^{4}$* and values of θ from the table of § 16 (2), all of which have been derived from the observations by Dobson[12]. We then obtain the following values :—

Latitude	5°	10°	20°	51° (Salisbury Plain)
h (metres)	2,530	1,760	1,240	803
$t_{\frac{1}{2}}$ (hours)	6·25	3·02	1·50	0·63
$t_{\frac{1}{10}}$ (hours)	20·8	10·0	5·0	2·1

$t_{\frac{1}{2}}$ is the period in which the amplitude is reduced to one half the initial value so that $e^{-\nu q_1^2 t} = 0{\cdot}69$; similarly $t_{\frac{1}{10}}$ is the period in which the amplitude is reduced to one tenth of its initial value. The table shows that in low latitudes the oscillations die out in a period of the order of half to one day ; in temperate latitudes they disappear in a few hours.

These conclusions apply only from the surface up to the height h to which the surface friction is assumed to be effective. A weakness of this treatment is that h has been arbitrarily selected, which would appear to introduce some degree of arbitrariness into the results. The further investigation required and the discussion of the numerical values arising from equation (18) are not considered appropriate to this report.

§ 17—THE FLOW OF AIR ACROSS THE ISOBARS

From the foregoing it is clear that the tendency for the wind to blow across the isobars is greater in the tropics than in temperate latitudes. Frictional effects give rise to a flow from higher to lower pressure, and the resulting transfer of air tends to diminish the pressure gradient ; isallobaric and accelerational effects may lead to a flow from higher to lower pressure or the reverse, the latter tending to increase the gradient. A complete investigation of the movement within the tropics must therefore take account of the variation of the pressure field due to the transfer of air. The problem is one of great difficulty, since the change of pressure at any level is due to the accumulation or loss of air integrated from that level to the upper limit of the atmosphere. It is insufficient to consider the flow across the isobars at one level or in one layer only, and the variation of pressure with height must be taken into account.

The results of § 15 and § 16 cannot be applied to this problem because they have been obtained on the assumption that the motion is independent of the co-ordinates x and y ; hence the horizontal divergence is everywhere zero, and the pressure cannot change as a result of the motion. The problem might be attacked on partly empirical lines, by estimating the changes of pressure gradient from the synoptic charts and expressing the variations in terms of circular or exponential functions of the time, or in any forms which lead to integrable differential equations.

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* Value for Salisbury Plain given by Sir Napier Shaw in "Manual of Meteorology", Vol. IV, London, 1931, p. 131.

§ 18—CONVERGENCE AND DIVERGENCE

The word " divergence " is, where the context permits, used in its mathematical sense to mean the general phenomenon of convergence and divergence.

(1) *General.*—Convergence and divergence present perhaps the most difficult and interesting of all the problems that confront the forecaster in the tropics. That they are of basic importance in the tropics, where they arise simply as characteristics of the field of flow, has already been indicated in § 6. It is the belief of the writer that many if not most of the frontal zones of the tropics are merely zones of convergence. There is however no obvious reason why convergence should be concentrated in narrow zones, and it is considered therefore that the effect may also operate over large areas and give rise to widespread bad weather ; divergence on the other hand is usually regarded as a fairly extensive phenomenon giving rise to fair or fine weather where it occurs.

The problem is to calculate the divergence from the field of horizontal flow, considered if possible in conjunction with the pressure field. It is sufficient for this purpose to assume that the air is incompressible ; the equation of continuity is then

$$\frac{\partial w}{\partial z} = -\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right),$$

which relates the vertical motion to the horizontal divergence. If further it is assumed that u and v do not vary with height in a layer from $z = z_1$ to $z = z_2$, we obtain on integration through the layer

$$w_2 - w_1 = -(z_2 - z_1)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right),$$

where w_2 and w_1 are the vertical velocities at the heights z_2 and z_1 respectively.

If the layer is next the surface so that $z_1 = 0$, $w_1 = 0$, we have

$$w_2 = -z_2\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right),$$

so that for such a layer convergence gives rise to an upward velocity and divergence to a downward velocity. For a layer not resting on the surface this does not hold, and the convergence or divergence determines only the difference between the vertical velocities at the upper and lower surfaces ; the actual velocities can only be determined by a step-by-step process for the layers, commencing with the lowest. In estimating the divergence it is necessary to consider the combined effect of all the layers.

(2) *Stream-line analysis.*—A little progress towards the solution of the problem has been made in some areas by mapping the wind field at different levels and drawing the instantaneous stream-lines. The wind direction at each station is shown by a short arrow and the speed is written in figures beside it ; the stream-lines are then drawn in freehand, an operation requiring care and judgment. At first only the directions of the main streams can be represented by this method, but with practice it is possible to draw the lines so that, roughly, their distance apart is inversely proportional to the speed of the wind. When this is done some of the stream-lines have free ends, appearing in areas of divergence and ending in areas of convergence. At best this is only a very rough method, and, in general, it may be said that the observations of wind are too thinly spaced to enable accurate stream-lines to be drawn. The areas of convergence and divergence have to be located by eye, and this can be done only approximately. In South Africa[14], as an aid to the drawing of stream-lines, preliminary charts are constructed showing isopleths of wind direction (isogonals) and of speed (isovels) ; the stream-lines can then be drawn more accurately than by using merely the raw data, but the problem of estimating the divergence is made very little easier.

A typical upper wind chart with stream-lines and the more obvious convergence zones is shown in Fig. 2. It is easy to pick out the convergence zones when they are associated with definite air streams meeting at a pronounced angle, but much more difficult when their detection depends upon the spacing, appearance and disappearance of stream-lines.

Apart from the location of areas of convergence and divergence, stream-line charts have other uses. The main one is the interpolation of upper winds in areas for which observations are not available, for, in the tropics this cannot be done on the basis of isobars. Another is the association of weather with characteristic flow patterns, the stream-lines filling a role similar to that of isobars in the higher latitudes, but up to the present the idea has not been much developed.

(3) *Mapping the field of divergence.*—An example will now be given of the calculation, from the observed winds, of the field of horizontal divergence, the same data being used as in Fig. 2, p. 23 The principle has been used in some tropical areas, notably in the Sudan by Solot[15], but the technique here described differs somewhat from his. It is considered that this method allows the data to be used to greater advantage than does the stream-line method, because that part of the work which depends upon estimation by eye is subject to less uncertainty. The work proceeds in steps as follows :—

(*a*) Resolve the observed winds into components u and v towards east and north respectively, and plot the values of u and v on separate charts (Figs. 3 and 4). Then draw isopleths of u and v. In the example, u and v are measured in kilometres per hour and the isopleths are drawn at intervals of 10 Km./hr.

(*b*) Calculate the value of the gradients of u and v at as many points as possible, selecting points on Figs. 3 and 4 where the isopleths give definite gradients which can be estimated with the minimum of ambiguity. At these points draw arrows perpendicular to the isopleths to show the gradient directions, and calculate the gradient magnitudes by measuring the intercepts of the isopleths on the arrows. In Figs. 3 and 4 the magnitude of the gradient in kilometres per hour per 100 kilometres is written beside each gradient arrow.

(*c*) Calculate the components of the gradients of u and v in the east and north directions respectively, thereby obtaining the values of $\partial u/\partial x$ and $\partial v/\partial y$. These are written, with the appropriate signs, beside the arrows in Figs. 3 and 4.

(*d*) On separate charts, Figs. 5 and 6, plot the values of $\partial u/\partial x$ and $\partial v/\partial y$ at the positions of the mid points of the gradient arrows of Figs. 3 and 4, and draw the isopleths.

(*e*) By gridding the isopleths of Figs. 5 and 6 plot the values of $\partial u/\partial x + \partial v/\partial y$ and draw the isopleths, Fig. 7.

In resolving into components in (*a*) and (*c*) it is sufficient to take the direction to the nearest 10 degrees. The work may be shortened by using a " components table " constructed for intervals of 1 unit in velocity or velocity gradient, and 10° in direction from 0° to 90°.

If the arrows in Figs. 3 and 4 are drawn for exactly the same points on the chart, the values of $\partial u/\partial x + \partial v/\partial y$ for these points may be obtained by simple addition. But it often happens that the most suitable points for measuring $\partial u/\partial x$ are not suitable for measuring $\partial v/\partial y$. In the gridding process (*e*) the intersections of the isopleths of $\partial u/\partial x$ and $\partial v/\partial y$ give a large number of points for mapping the field of $\partial u/\partial x + \partial v/\partial y$.

Fig. 8 shows the precipitation reported at about 1200 G.M.T. on the day for which Fig. 7 was constructed. The following are points of interest in regard to the main precipitation and convergence areas, indicated by letters in Fig. 8.

A. Kenya and north Tanganyika. There is a marked area of convergence on Fig. 7 and widespread precipitation is shown on Fig. 8. This however might have been deduced from the stream-line chart alone.

B. North Kenya and Somaliland. A few scattered showers perhaps associated with the north-eastward extension of the convergence area A. From the stream-line chart convergence might have been expected in the neighbourhood of B owing to the rapid decrease of wind between the Gulf of Aden and North Kenya. The air was however too dry to give much precipitation.

C. Eritrea and Red Sea. Moderate convergence but the air was too dry for much precipitation.

D. Palestine and Transjordan. Marked convergence and fairly general precipitation associated with a cold front.

E. Eastern Madagascar. Divergence, but many reports of precipitation in sight. These were orographic showers over the high mountains a few miles inland, and are not unusual in almost any meteorological situation in this area.

F. South-westward of Madagascar. Marked convergence associated with a cold front which moved into the area from the south-west. No observations were available over the sea to show whether precipitation occurred or not.

G. South Africa. The convergence was associated with the rapid spreading of an intense anticyclone in cold air from the south-west. The air was very dry so that there were only a few reports of precipitation.

Figs. 2 to 7 refer only to the winds at 2 Km., and are therefore representative only of the layer from 1·5 to 2·5 Km. For a complete picture of the field of divergence the same computation should be carried out for the winds at the other levels and the total divergence calculated by adding the results.

The process outlined above leads to an estimate of the divergence at the time of observation of the winds. It is a problem for investigation whether the variation of the divergence with time could be estimated by the methods of § 15, using the observed winds and the pressure distribution.

(4) *Units of divergence*.—The units of divergence have been discussed by Kindle[16] from whose paper the following remarks are mainly taken. The dimensions of divergence are $M^0L^0T^{-1}$ in a system of units in which the unit of length is the same for the velocity as for the velocity gradient. In the C.G.S. system the unit is the sec.^{-1}, and the values of divergence ordinarily significant range from 10^{-8} to 10^{-4} sec.^{-1}. For practical purposes a more convenient unit is the kilometre per hour per 100 kilometres; it is equal to $2 \cdot 78 \times 10^{-6}$ C.G.S. units, and has been used in constructing Figs. 3 to 7. The following table, showing the ranges of values of the divergence, in C.G.S. units, and the type of weather with which they are usually associated, is taken from Kindle's paper. The equivalent values in kilometres per hour per 100 kilometres are shown in the second column.

Divergence		
C.G.S. units	Km./hr./100 Km.	Approximate weather
-10^{-4}	-36	Heavy precipitation
-10^{-6} to -10^{-5}	$-\ 3\cdot6.10^{-1}$ to $-3\cdot6$	Light precipitation
-10^{-8}	$-\ 3\cdot6.10^{-3}$	Very little weather
$+10^{-8}$	$+\ 3\cdot6.10^{-3}$	Very light subsidence
$+10^{-6}$ to $+10^{-5}$	$+\ 3\cdot6.10^{-1}$ to $+3\cdot6$	Moderate subsidence

This table applies to the tropics, the third column being based on the average values of humidity in those regions.

According to this table the maximum values of the convergence in Fig. 7 correspond to something intermediate between light and heavy precipitation.

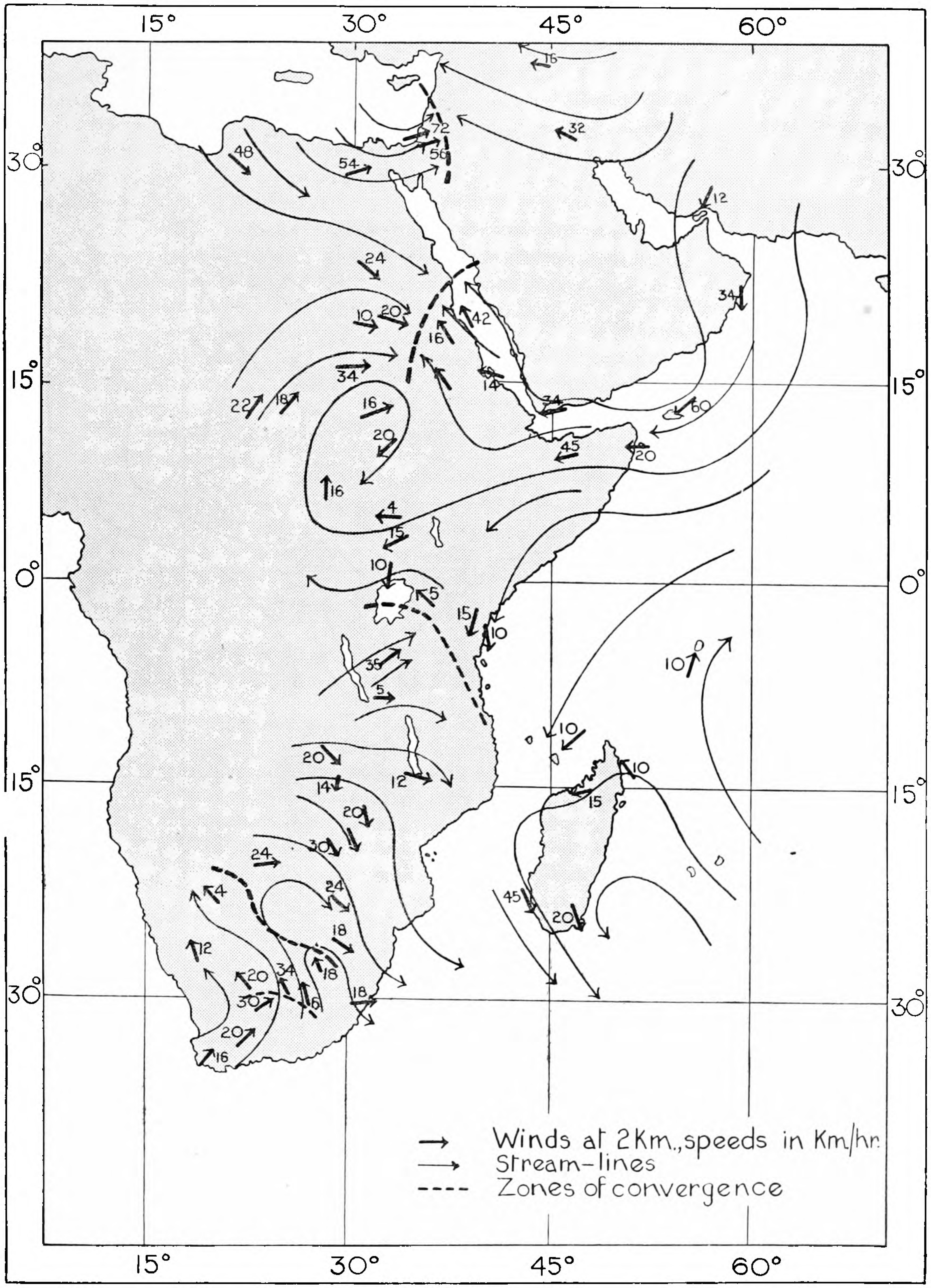
15°
30°
45°
60°
30°
15°
0°
15°
30°
Winds at 2 Km., speeds in Km/hr.
Stream-lines
Zones of convergence

FIG. 2—STREAM-LINES AND ZONES OF CONVERGENCE FOR THE MORNING OF NOVEMBER 24, 1944

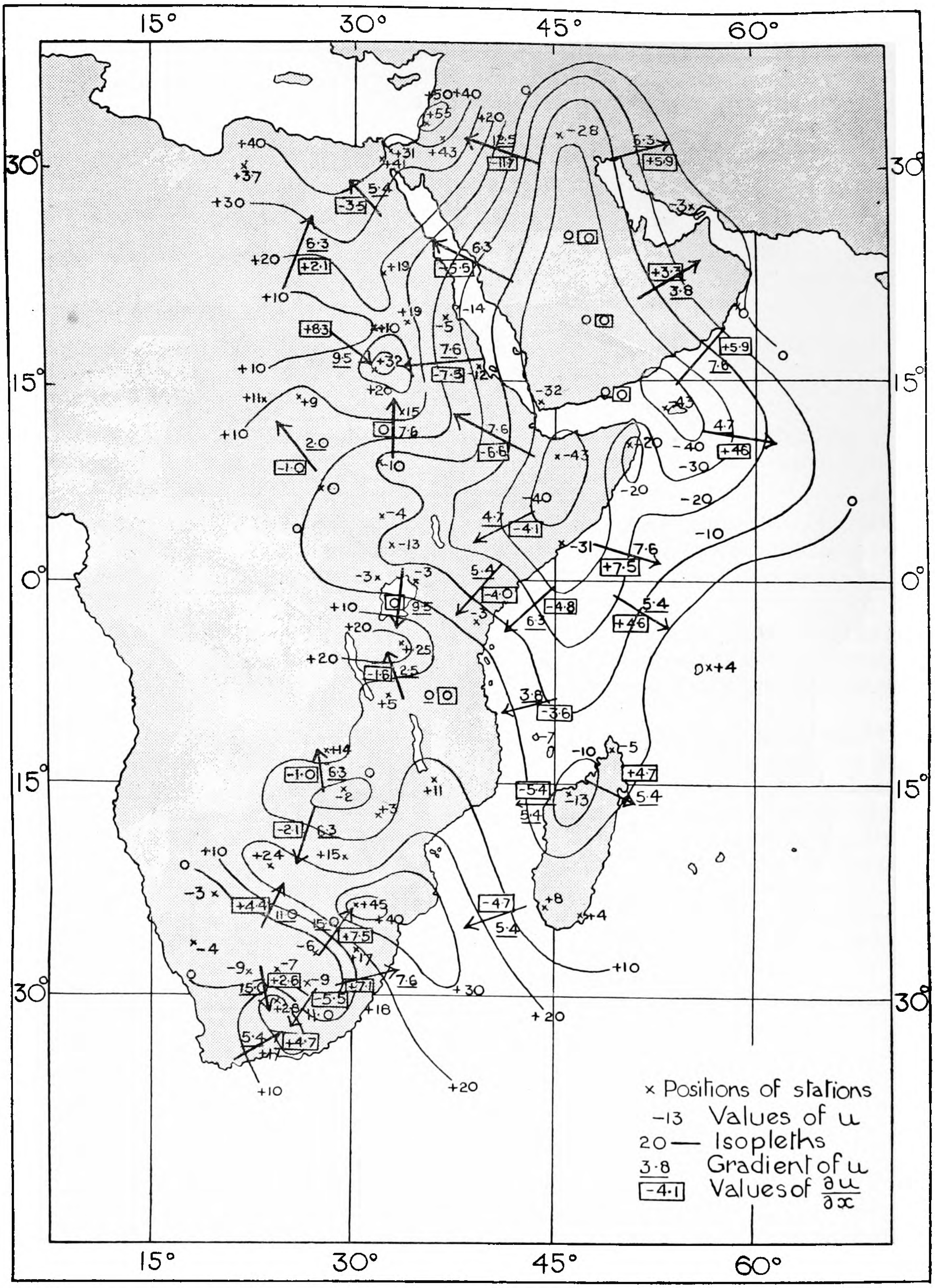
15°
30°
45°
60°
30°
15°
0°
15°
30°
+50
+40
+55
+20
+40
+37
+31
+41
+43
12.5
-11.7
-28
6.3
+5.9
5.4
-3.5
+30
-3
6.3
+2.1
+20
+19
6.3
-5.5
+3.3
3.8
+10
0
+19
-5
-14
+8.3
+10
9.5
+32
7.6
-7.5
-12
+5.9
7.6
+10
+20
+11
+9
15
-32
-43
4.7
+4.6
+10
2.0
-1.0
0
7.6
-10
7.6
-6.6
-43
+20
-40
-30
-20
-20
-4
-40
4.7
-4.1
-10
-13
-31
7.6
+7.5
-3
-3
5.4
-4.0
+10
9.5
-3
-4.8
6.3
5.4
+4.6
+20
+25
+20
-1.6
2.5
+5
+4
3.8
-3.6
-7
-10
-5
+14
+4.7
-1.0
6.3
-2
+11
-5.4
-13
5.4
5.4
+3
-2.1
6.3
+10
+24
+15
-3
+4.4
11.0
15.0
+45
+40
-4.7
+8
+4
5.4
+7.5
-6
+17
-4
-9
-7
+2.6
-9
+7.1
7.6
15.0
+28
-5.5
+10
+18
+30
+20
11.0
5.4
+4.7
+17
+10
+20
× Positions of stations
-13 Values of u
20— Isopleths
3.8 Gradient of u
-4.1 Values of ∂u/∂x

FIG. 3—ISOPLETHS OF WIND COMPONENT *u* (IN KM./HR.) FOR THE MORNING OF NOVEMBER 24, 1944

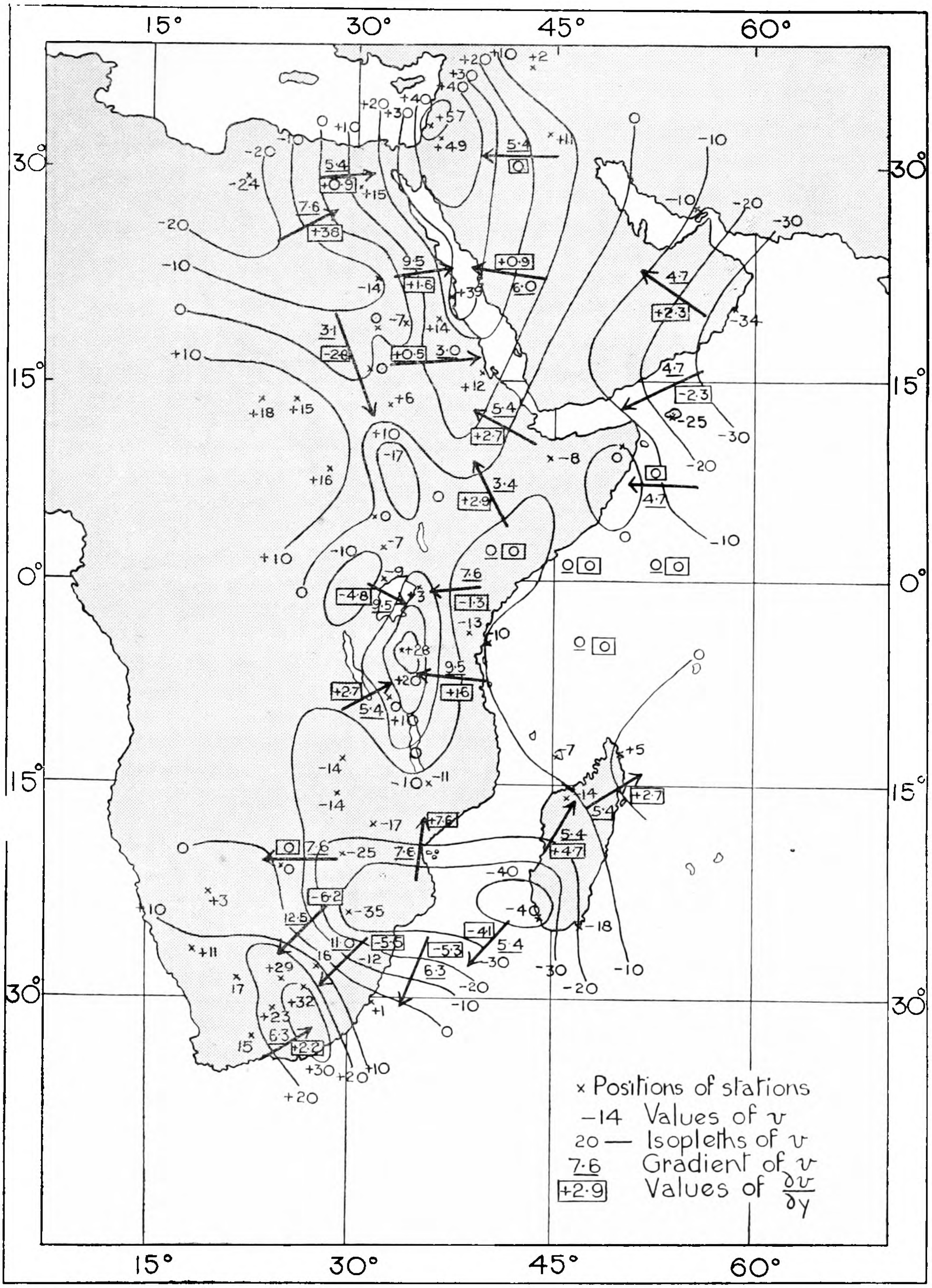
15°
30°
45°
60°
30°
15°
0°
15°
30°
× Positions of stations
−14 Values of v
20 — Isopleths of v
7·6 Gradient of v
+2·9 Values of $\frac{\partial v}{\partial y}$
+20
+10
+2
+30
+40
+57
+49
5·4
0
+11
−10
−20
−30
−24
+0·9
+15
7·6
+38
9·5
+1·6
+39
6·0
4·7
+2·3
−34
−14
−7
+14
3·1
−2·8
+0·5
3·0
+12
−2·3
−25
+18
+15
+6
5·4
+2·7
−8
+16
−17
3·4
+2·9
−7
−4·8
−9
3
−1·3
−13
+28
+1·6
+2·7
−11
+5
−14
−17
+7·6
−25
+4·7
−40
+3
−6·2
−35
12·5
11·0
−5·5
−5·3
−41
6·3
−12
+11
+29
+32
+23
17
15
+2·2
+1
+30
−18

FIG. 4—ISOPLETHS OF WIND COMPONENT v (IN KM./HR.) FOR THE MORNING OF NOVEMBER 24, 1944

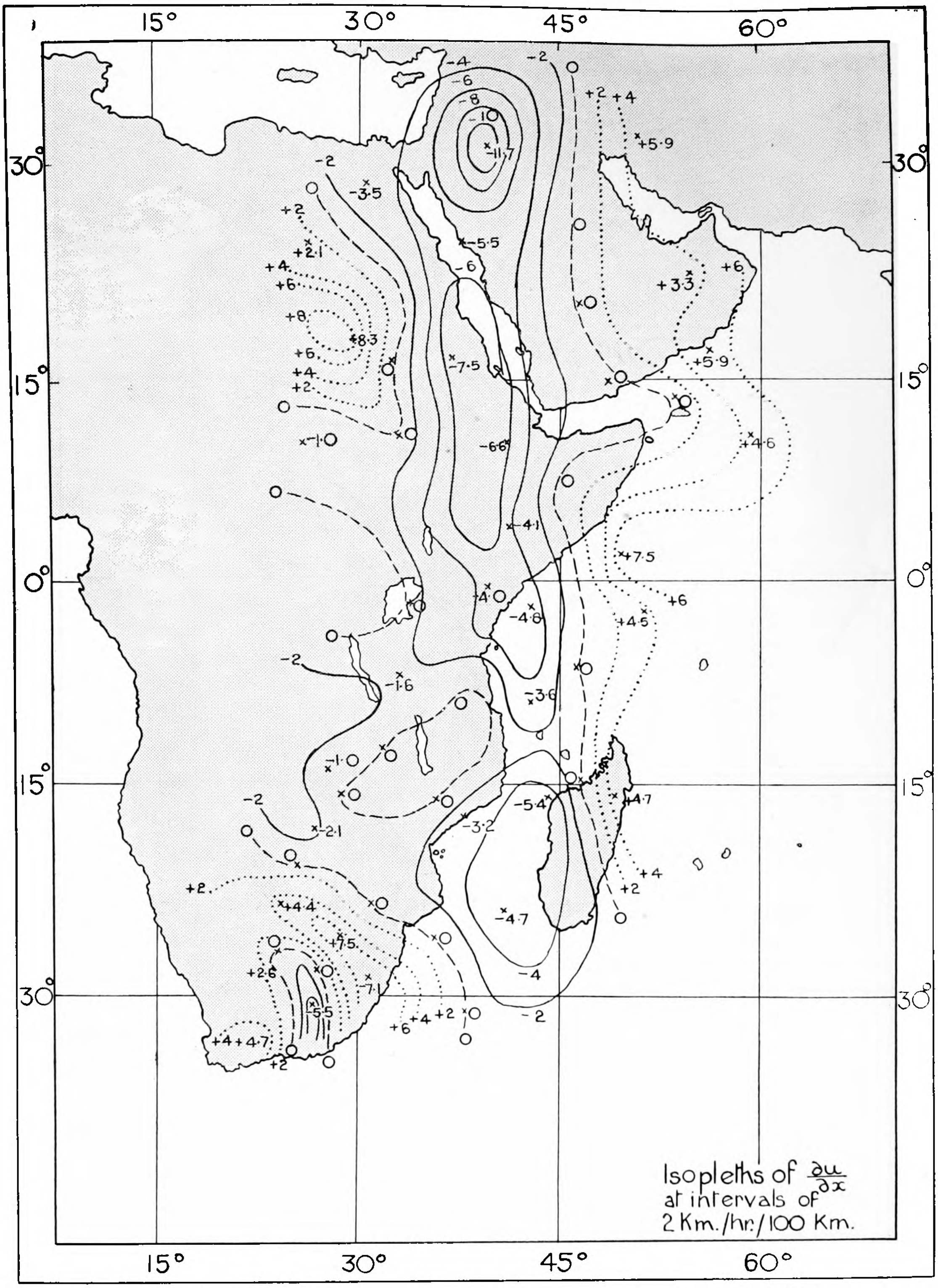
15°
30°
45°
60°
30°
15°
0°
15°
30°
-4
-6
-8
-10
-11.7
-2
+2
+4
+5.9
-3.5
-5.5
-6
+2.1
+3.3
+6
+8.3
-7.5
+5.9
-1.0
-6.6
+4.6
-4.1
+7.5
+6
-4
-4.8
+4.5
-1.6
-3.6
-1.0
-5.4
+4.7
-2.1
-3.2
+4
+4.4
-4.7
+7.5
+2.6
-7.1
-5.5
+4
+4.7
+6
-2
Isopleths of $\frac{\partial u}{\partial x}$ at intervals of 2 Km./hr./100 Km.

FIG. 5—ISOPLETHS OF $\partial u/\partial x$ FOR THE MORNING OF NOVEMBER 24, 1944

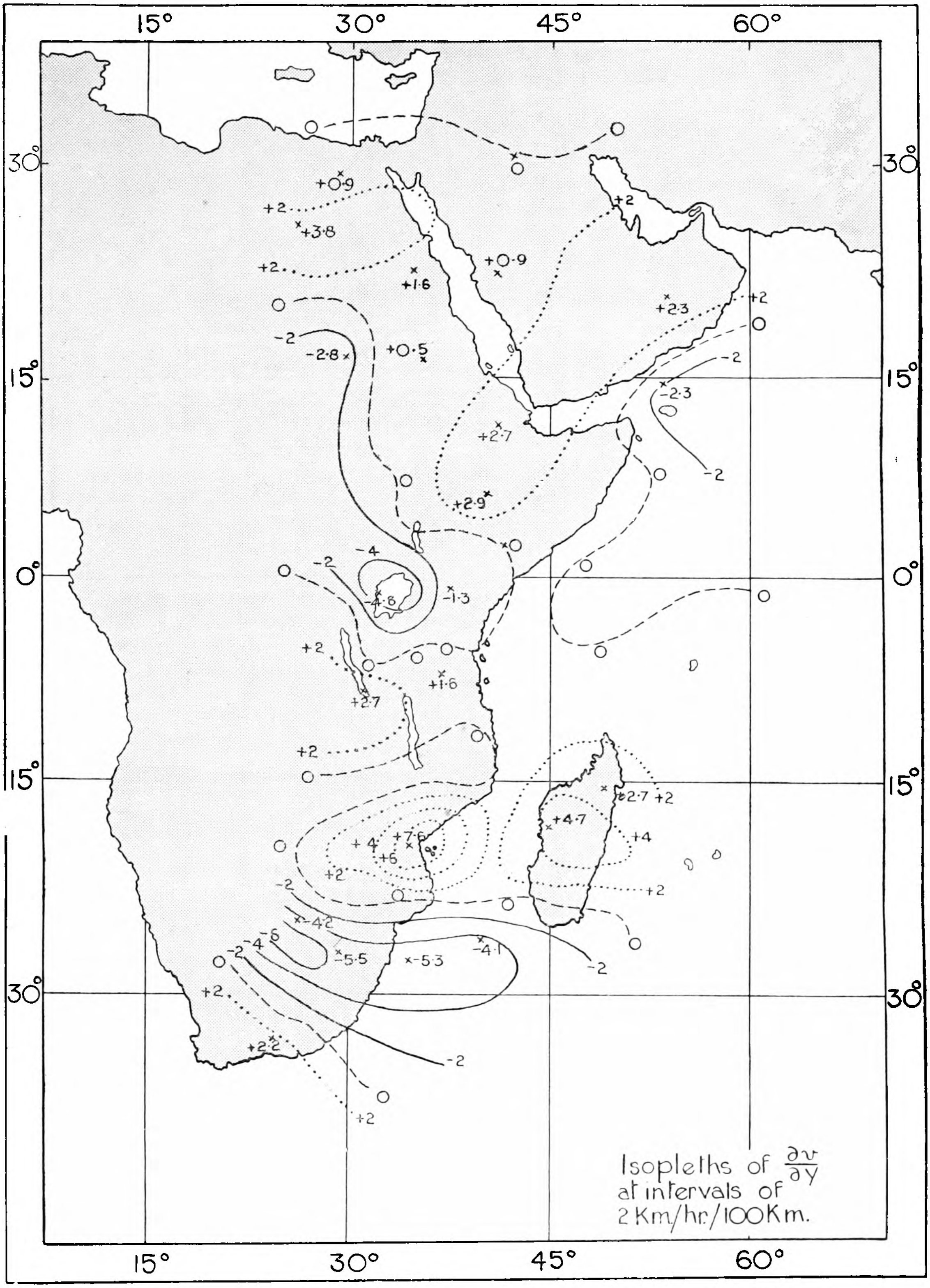
15°
30°
45°
60°
30°
15°
0°
15°
30°
+0·9
+2
+3·8
+2
+1·6
+0·9
+2
+2·3
+2
-2
-2·8
+0·5
-2
-2·3
+2·7
-2
+2·9
-4
-2
-4·8
-1·3
+2
+1·6
+2·7
+2
+2·7
+2
+4·7
+4
+4
+7·6
+6
+2
+2
-2
-4·2
-6
-4
-2
-5·5
-5·3
-4·1
-2
+2
+2·2
-2
+2
Isopleths of $\frac{\partial v}{\partial y}$
at intervals of
2 Km/hr/100Km.

FIG. 6—ISOPLETHS OF $\partial v/\partial y$ FOR THE MORNING OF NOVEMBER 24, 1944

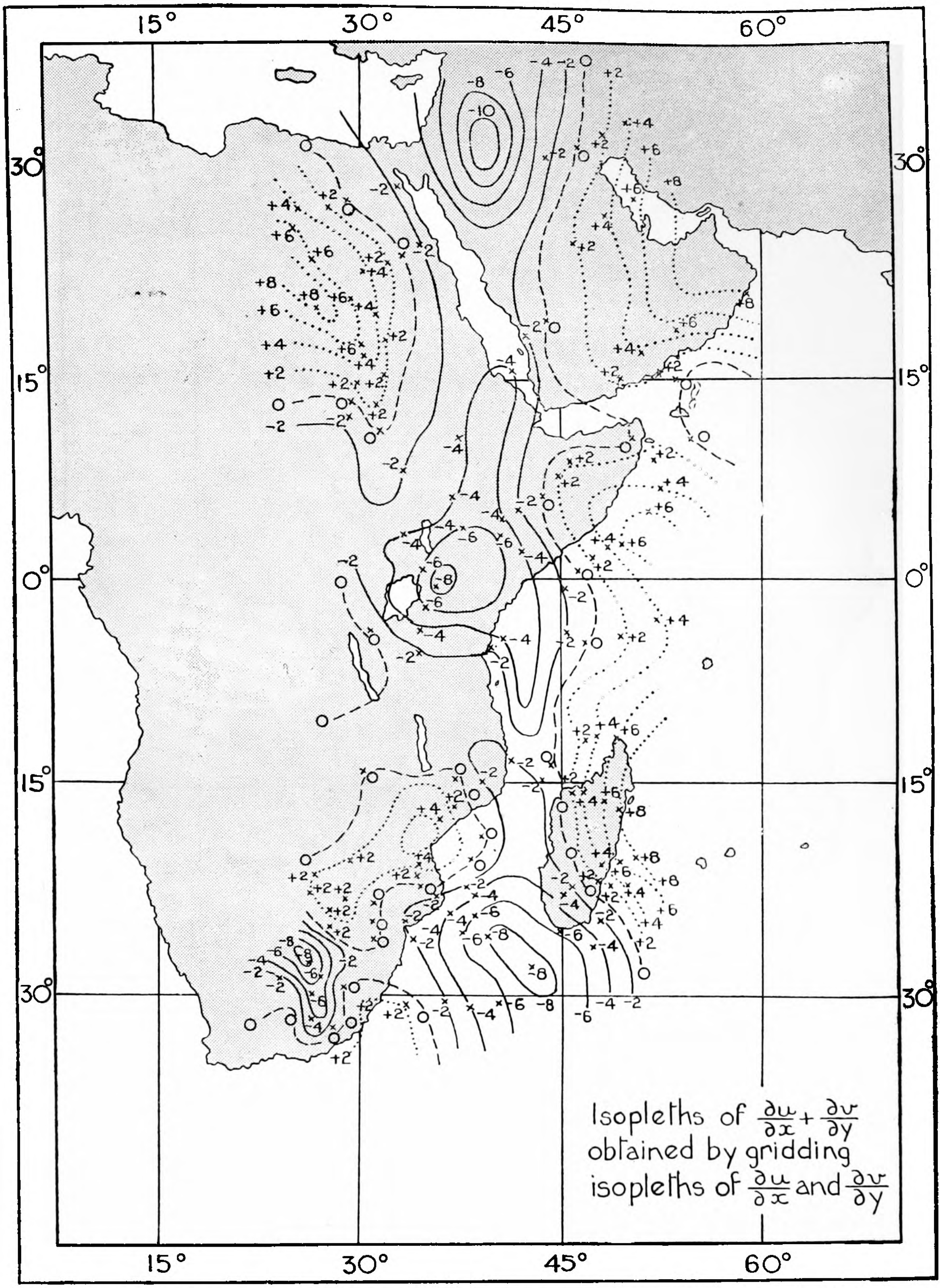
15°
30°
45°
60°
30°
15°
0°
15°
30°
Isoplefhs of $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}$ obtained by gridding isopleths of $\frac{\partial u}{\partial x}$ and $\frac{\partial v}{\partial y}$

FIG. 7—ISOPLETHS OF $\partial u/\partial x + \partial v/\partial y$ FOR THE MORNING OF NOVEMBER 24, 1944

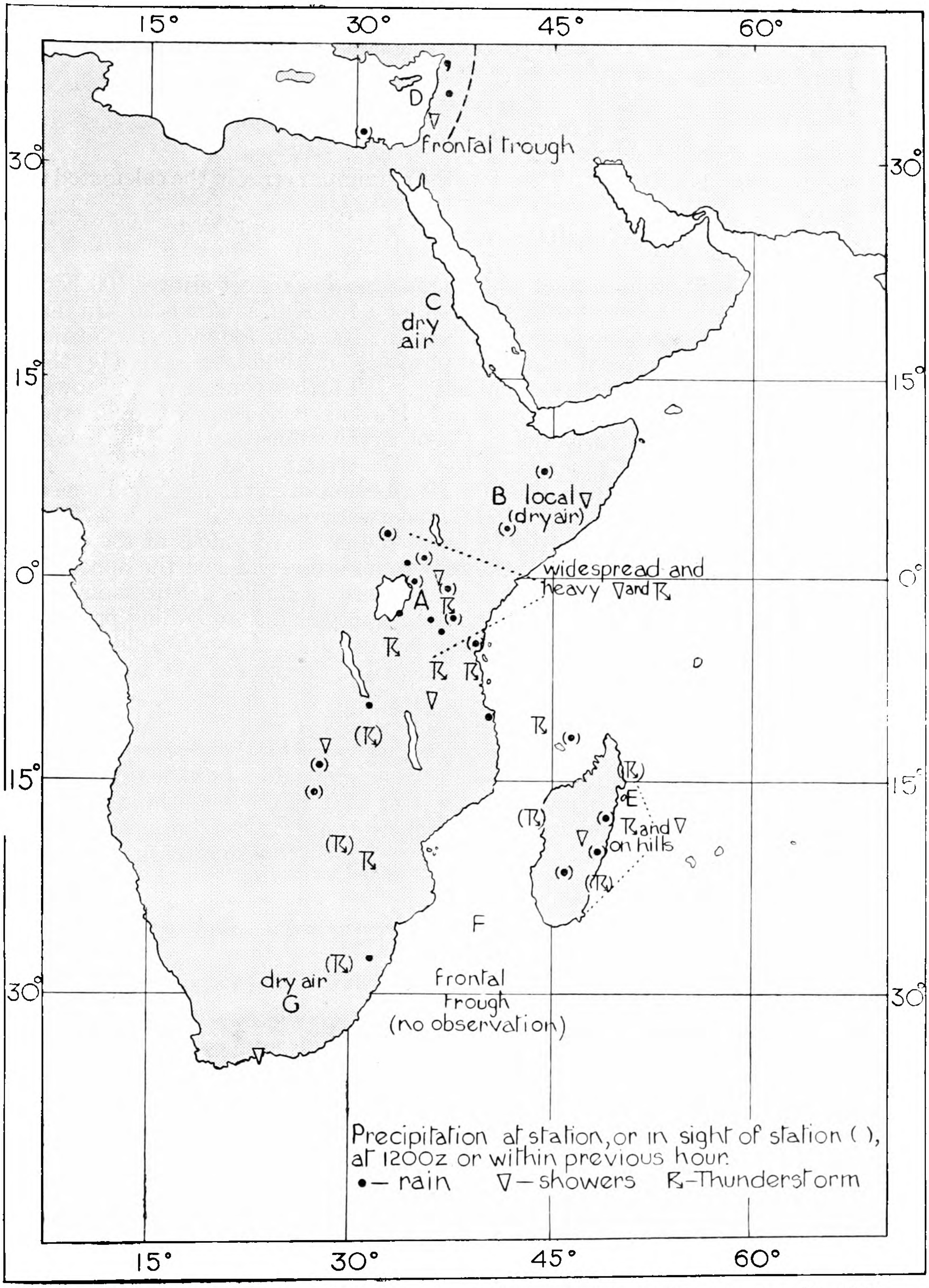
15° 30° 45° 60°
30°
15°
0°
15°
30°
D
frontal trough
C
dry air
B local ∇
(dry air)
widespread and
heavy ∇ and ⟆
A
E
⟆ and ∇
on hills
F
frontal
trough
(no observation)
dry air
G
Precipitation at station, or in sight of station ( ),
at 1200z or within previous hour.
• — rain ∇ — showers ⟆ — Thunderstorm

FIG. 8—PRECIPITATION AT 1200 G.M.T. OR WITHIN PREVIOUS HOUR NOVEMBER 24, 1944

(5) *Errors arising from inaccurate wind measurement.*—Suppose that the wind measurement is subject to a maximum error δV, and that the velocity gradients are calculated from observations at stations uniformly spaced in a square pattern the side of each square being d hundreds of kilometres. The maximum error in

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y} \text{ is then } \frac{4\delta V}{d}.$$

Now δV may be taken as 5 Km./hr. Therefore the maximum error in the calculated divergence is

$$\frac{20}{d}\text{ Km./hr./100 Km.}$$

With the existing distribution of pilot balloon stations d is almost always 100 Km. or more, and the maximum error in the divergence, calculated for a 100-Km. square is 20 Km./hr./100 Km. This compares with the maximum value of 8 Km./hr./100 Km. found in the example in (3) p. 21 from which it would appear that the proposed method for mapping the field of divergence is subject to too large errors to be of any use. Further consideration shows, however, that this conclusion is incorrect. As far as u only is concerned, the maximum error in $\partial u/\partial x$ calculated for a 100-Km. square can arise only if the observed values of u have the maximum error at the two upwind corners of the square and the same error but of opposite sign at the two downward corners, with simultaneously a similar distribution of errors in v; and generally, for a square of side d hundreds of kilometres and a 100-Km. network of observing stations, the maximum error, $20/d$ Km./hr./100 Km., can only arise if, for u, the observed values at the $(d+1)$ points on the upwind side of the square each have the maximum error, while on the opposite side they have the same error but of opposite sign, with simultaneously a similar distribution of errors in v. But in fact the errors in u and v would be random ones to which a smoothing process would be applied in calculating the mean values along the respective pairs of opposite sides of the square, and this would in general reduce the error in the calculated divergence to a small fraction of its maximum possible value.

We therefore conclude that, though the maximum possible errors in the calculated divergence are of the same order of magnitude as the maximum values found for the divergence in the example of (3) p. 21, the distribution of divergence found in the example does in its main features at least approximate to the true distribution, owing to the fact that in the smoothing process inherent in the drawing of the isopleths of u and v observational errors are largely eliminated. But whether the method is a useful one or not can only be decided by trial in a considerable number of particular instances.

PART III—UPPER AIR AND CONVECTIONAL PHENOMENA

§ 19—CO-ORDINATION OF UPPER AIR OBSERVATIONS

Upper air temperature observing stations are at present so widely spaced in the low latitudes that, except over certain limited areas such as India and the southern part of the U.S.A., it is not possible to construct upper air charts and atmospheric cross-section diagrams as in temperate latitudes. Though the observations are of great value individually they cannot yet, in general, be co-ordinated. It is unfortunate that, since the relation between the wind and the pressure distribution in the tropics is not a simple one, the observations of upper winds afford little assistance in constructing thickness lines and contours of the isobaric surfaces, which have therefore to be based almost entirely upon the heights and thicknesses computed from the available upper air temperature reports. A large increase in the number of radio-sonde stations in the tropics is therefore necessary, and if real understanding of the atmospheric circulation is to be obtained the network must be at least as dense as that of temperate latitudes. If, as seems likely, it is not possible to establish a network of this general density, it would probably be better, at least in the initial period, to concentrate the available radio-sondes in fairly close groupings over limited areas rather than to spread them thinly over the whole of the tropical regions, in order to see whether there are significant variations of upper air conditions over short distances, which the more widely spaced stations would fail to detect. An outstanding problem for investigation is that of the circulation above the subtropical anticyclones and the equatorial low-pressure belt, which are maintained by high-level convergence and divergence respectively.

In India, where there is a fairly close network of upper air temperature observing stations, some work has been done on the construction of charts of upper air contours. In a recent unpublished paper by R.A.S. Ratcliffe, it has been concluded that there is a tendency, as might be expected, for bad weather to be associated with " cold pools " between the 1000- and 500-mb. levels ; but the problem of forecasting the development and movement of the cold pools has still to be investigated. In temperate latitudes use is made of the concept of the geostrophic transport of thickness lines[17] in this connexion, but this cannot be done in low latitudes where geostrophic conditions very rarely obtain. It might be possible, however, to estimate the movement of the cold pools, using stream-line charts, if necessary, in conjunction with the equations (6) of § 15. Some unpublished work by Schove, in west Africa, shows that " cold troughs " moving from east to west between Nigeria and Sierra Leone are fairly common during the winter months when constant NE. winds prevail, indicating that under steady conditions the cold pools may be expected to move with the general currents.

The effect of high ground on the contours of the upper air pressure surfaces has an important practical application in connexion with altimeter corrections for aircraft. It has been examined, very roughly, in east Africa by working out the mean heights of the 500-mb. surface for three stations, Mombasa at sea level, Nairobi at 5,400 ft., and Soroti (Uganda) at approximately 4,000 ft. These stations lie almost in a straight line, of total length about 550 miles, with Nairobi about midway between the other two. Over the limited period examined it was found that the average height above sea level of the 500-mb. surface was at Nairobi about 600 ft. and at Soroti about 500 ft. above that at Mombasa, suggesting that the upper level pressure surface tends, so to speak, to rise and fall with the ground surface. This is apparently because over the high ground the insolational heating is applied at a considerable height above sea level, and the lowest few thousand feet of air are, during the day, a good deal warmer than the air at the same level over the sea or plains. From this it appears that high ground may have a marked influence on the air flow not only at its own level, but for a considerable height above. The high upper level pressure over the east African plateau, together with the effects of the plateau itself are probably the cause of the rapid change in direction of the SE. monsoon along the east African coast, which is frequently S. at Mombasa (4° S.) and SW. at Mogadishu (2° N.).

§ 20—CONVECTIONAL PHENOMENA

Most of the rainfall of the tropics is convectional in origin, or, when primarily due to other causes, it is increased owing to convectional effects. Individually, particularly at inland stations, the tephigram is the greatest single aid to local forecasting ; in east Africa it is of greater assistance than the synoptic chart, especially when forecasting the day's weather from the early morning observations. A combination of lapse rate greater than the saturated adiabatic, with high humidity, at least up to the freezing level is required for the outbreak of general precipitation, though light precipitation is often observed when the cloud tops do not reach the freezing level. On many occasions when the lapse rate is quite unstable the weather remains fair because there is insufficient moisture for precipitation ; the importance of accurate values of humidity in the upper air cannot therefore be over-emphasised. Because of the relatively large amounts of water vapour condensed for a given degree of cooling at the high temperatures prevailing, convectional clouds are very dense, and the precipitation from them heavy in comparison with temperate latitudes ; on the other hand, many cumulus clouds which would give precipitation in temperate latitudes do not do so in the tropics because they do not develop to the high altitude of the freezing point.

In forecasting convection the usual criteria for instability are applied to the tephigram, but it has been found better to use the mean water-vapour content in the lowest 100 mb. than to use merely the water-vapour content of the surface air ; i.e., it should be assumed that the ascending air parcel is a thoroughly mixed sample, representative of a thick layer of air. It has hitherto been assumed that the effect of thorough mixing is to bring about a uniform distribution of potential temperature, and for this reason a curve of mean potential temperature from the surface up to various levels has been used in conjunction with a curve of mean water content for the same layers to find the condensation level for turbulence cloud. In view however of a recent paper by Priestley and Swinbank[18], which shows that, in addition to ordinary mechanical turbulence which transfers heat in the direction of decreasing potential temperature (normally downwards), there is an effect due to convective turbulence, which arises from buoyancy effects due to small-scale horizontal variations of temperature, and which may transfer heat upwards, the idea of turbulent mixing applied in this way to the tephigram requires further investigation.

On many occasions in east Africa the ascent curves indicated lapse rates slightly in excess of the saturated adiabatic, and there was enough moisture present to lead to convection if the ordinary parcel-method criterion held good, but precipitation did not usually follow. This was attributed to the stabilising effect of the descending environment, as discussed by Bjerknes[19]. It was found possible to allow for this very roughly, by assuming that the whole of the environment was lowered by an amount corresponding to an increase of pressure of 30 mb. A new temperature curve was constructed by drawing, from each of a number of points on the original, a dry adiabatic to the pressure line higher by 30 mb. The new environment curve drawn through the points thus obtained gave a better estimate of convection than the original curve. It must however be emphasised that the range of 30 mb. was purely empirical.

If the lapse rate is markedly unstable and the air moist, convection follows the normal diurnal sequence, reaching its peak in the late afternoon, and the resulting precipitation is often widespread, the individual storms merging to form a continuous rain area. With slight to moderate instability the afternoon convection is often restricted to the hills, and over the plains and plateau areas there is then little cloud, presumably owing to the compensating subsidence. During the night, however, the rain frequently extends to the plains also. It is thought that the precipitation over the hills cools the lower layers of air there and a katabatic outflow is set up, which, reaching the plains, undercuts and starts convection in the air which has been made unstable by the full day's insolation. Grimes[20] has mentioned similar effects in India and Burma, and has also remarked that in the Ganges Valley the outflowing cold air from below a thunderstorm causes the storm to be regenerated in the air surrounding it, giving rise to an appearance of continuous movement of the storm. Precipitation at night in convective situations may also be increased owing to the instability arising from radiation from the cloud tops, and also by the delayed realization of the latent instability brought about by the insolation of the preceding day in the manner described by Dexter[21] ; further investigation is required into these processes. Another feature of interest is that in a slightly stable situation, the cumulus cloud which forms during the forenoon decreases in the afternoon as warmer and drier air from near the surface takes part in the convective process. Finally there is the question of forecasting the low cloud or fog at dawn from the tephigram of the previous evening ; no success has yet been achieved in this problem since it is not known how the temperature and humidity in the lowest layers vary during the night. The investigations into turbulent and radiative cooling at night at present being carried out in Great Britain should throw much light on this problem.

§ 21—SYNOPTIC ASPECTS

The tropical regions are characterised by the absence of well defined changes of air-mass type, and therefore the markedly characteristic tephigrams, which are associated in temperate latitudes with the various air masses, are not much in evidence. Instead, the tephigrams tend, on the whole, to be characteristic of the locality and season, and for the most part the day-to-day fluctuations are small, though, as will be mentioned below, they are important in certain localities. Experience in east Africa and the western Indian Ocean indicates that there are four main types of tephigram to which brief consideration will now be given.

(i) Over the sea in the trade-wind belts, the tephigram usually shows an inversion or isothermal layer, outside which the lapse rate is approximately saturated adiabatic ; the air is moist below the inversion, but dry above. The height of this inversion, or more significantly the depth of the moist layer, increases from about 4,000 ft. in latitude 20–25° S. to 10,000 or 15,000 ft. in latitude 10–15° S., and nearer the equator the inversion is usually not found at all. With a shallow moist layer only small cumulus cloud are formed, but these develop into large cumulus and cumulonimbus where the moist air is deep ; hence the depth of the moist layer is a useful guide in forecasting. At Mauritius (20° S.) and in south Madagascar (25° S.) the inversion or isothermal layer sometimes disappears completely with a surge of cold air from S. or SW., a considerable fall of temperature occurring in the upper levels without much change at the surface, suggesting the north-eastward movement of an upper cold front, § 25 (2). In lower latitudes very little change of upper air temperature has been observed in association with these surges.

(ii) Over the land in the outer parts of the tropical regions the tephigrams are characterised by extreme dryness. On hot days the lapse rate becomes dry adiabatic up to 12,000 or 15,000 ft., and, though violent bumpiness and sometimes duststorms accompany these conditions, precipitation is rare.

(iii) Over the sea in the equatorial low-pressure belt the tephigrams show a lapse rate about equal to or slightly greater than saturated adiabatic and moist air at all levels, which conditions persist almost unchanged from day to day. Wave-like perturbations moving from an easterly direction, § 25 (1), in this area appear to have little or no effect on the ascent curves, which are therefore of little assistance in detecting these disturbances.

(iv) The Nairobi ascents have provided a good series of data for an inland station almost on the equator, but they exhibit some peculiar features, and it is not known to what extent they are typical of equatorial stations in other parts of the world. Though in general the day-to-day changes of upper air temperature are small, there are sometimes large and rapid changes at the high and medium levels, a moist unstable distribution becoming transformed into a dry stable one or *vice versa* within 24 hours. So far it has not been possible to forecast these changes. It is difficult to see how they could be brought about by advection ; nevertheless, this should be investigated by endeavouring to trace back the air at each level to its source region, using streamline charts constructed at as frequent intervals as the observations permit. A more likely explanation, however, is that the changes are brought about by the vertical motion resulting from horizontal convergence or divergence, for widespread rainfall is invariably accompanied by unstable moist ascent curves. It is important, therefore, that the effect of these on the lapse rate and humidity in the upper air be investigated. To this end the method of calculating the divergence discussed in § 18 might be used ; and Austin[22] has shown how to calculate the stretching or contraction of an air column from the divergence ; the associated precipitation can then be assessed if the original humidity distribution in the upper air is known.

PART IV—FRONTAL ZONES AND DISCONTINUITIES, TROPICAL STORMS

§ 22—GENERAL

Though frontal ideas have become well established in relation to the weather of temperate latitudes and form the basis of analysis and forecasting in those regions, their extension to the tropics is still only tentative. Theoretical considerations supported by climatological observations have in the past led to the generally accepted view that, broadly speaking, the air of the tropics consists of a vast, almost homogeneous, warm, moist mass, within which the only factors operative for the production of cloud and rain are radiation and orographic and convectional effects, modified by the diurnal variation of insolation. The area between latitudes 30° N. and S. comprises one half of the earth's surface, and the greater part of it consists of ocean, the variations of whose surface temperature are, in the main, slight and gradual. Over this extensive, almost uniform surface the air movement is comparatively slow, often very slow, and not only is it usually impossible for appreciable discontinuities of air temperature to be set up in the region, but those from outside tend to be quickly smoothed out. The whole air mass acquires a high temperature, and, in the lower layers at least, a high moisture content. It would appear therefore that when this air is subject to convection resulting from its movement over heated land areas, to orographic uplift when it meets the numerous and substantial mountain barriers and to the uplift due to convergence between the trade winds of the northern and southern hemispheres, the ensuing copious rainfall and heavy cloud formation could be readily explained in terms of air-mass properties without having recourse to frontal ideas.

It is, however, a matter of observation that the weather of many parts of the tropics does not always follow a regular diurnal sequence, nor does it have a fixed distribution from day to day, as it would if it were the result of the action of the above processes on a homogeneous air mass. One day is rarely the exact counterpart of its predecessor, though there is on the whole far less variation than in higher latitudes. For example, in the Kenya highlands during the SE. monsoon, thick low cloud with rain or drizzle commencing during the night may on some occasions persist until midday or later ; on others they may disappear in the early part of the forenoon, while on many days during this season the weather remains continuously fair. In India the tendency of the SW. monsoon to occur in " bursts " of increased wind associated with much cloud and heavy rainfall, followed by quiet relatively fair periods has been described by many writers. The present writer's experience in flights over the western Indian Ocean has shown the existence of belts of disturbed weather, which, there is good reason to believe, change their position and intensity from day to day. Reports from other regions also support the view that superposed on the general constancy of tropical weather there is a variable element, ill defined but none the less real. It is natural to attempt to relate this changeability of weather to horizontal variations of characteristics within the air mass, and in particular to consider whether these variations can ever become so sharp as to constitute narrow transition zones or possibly fronts.

Fronts may be expected to occur within the tropics either as a result of their intrusion from higher latitudes or of frontogenetical processes within the tropical air mass. In the first of these processes the polar front is swept into the tropics while an intense anticyclone builds up in the cold air behind it. To begin with the front exhibits a marked discontinuity of temperature, and, though on moving into the tropics it is subject to frontolysis on account of surface influences and subsidence of the cold air, it should be possible, given a sufficiently close network of observing stations, to achieve continuity in tracing it from chart to chart. On account of the masking effect of diurnal variation and other influences the surface observations are in many cases non-representative; the effect on pressure tendencies has already been mentioned, §6; at coast stations the general wind flow is much affected by pronounced sea breezes, and day-time temperatures depend almost entirely upon the cloudiness. In the second process there is little or no difference of temperature and humidity between the front-forming air masses, but the weather is caused by the uplift resulting from convergence between them ; the front may be regarded as dynamical rather than thermal. The formation and persistence of a front of this type depends upon the field of flow ; the zone of convergence is a feature simply of the flow pattern and it has no permanent existence in the sense of being associated with particular particles of air or with the boundary between two distinguishable air masses. Examples of convergence zones are the intertropical trough between the trade winds of the northern and southern hemispheres, and the surges of the trades. It may happen that the thermal and dynamical causes of fronts are simultaneously present ; for example the equatorward movement of an old polar front is often associated with a surge in the general current, and it is to be expected that under these circumstances the front will be more active than it normally is.

A third type of zonal disturbance is the wave-like perturbation or trough, lying more or less at right angles to the general air stream and appearing as a line of small sinuosities in the isobars. Little is known of the causes of origin of such troughs, but they appear to be similar to the minor perturbations which travel along " straight isobars " of temperate latitudes.

In recent years, as a result of the increased attention given to tropical meteorology during the war and the improved synoptic reporting networks, fronts in low latitudes have been the subject of study of most of the tropical meteorological services. Though differing on points of detail most of the papers which have been written are in agreement on general principles. The present state of knowledge is best summarised in a paper published by the University of Chicago, Institute of Tropical Meteorology.[23]

§23—CLASSIFICATION OF FRONTS

Early in 1945 an American committee deliberated on the standardization of tropical nomenclature and received the benefit of J. Bjerknes' participation. This Committee recommended the following definitions. Comments on them have been inserted in brackets.

Semi-permanent features

Intertropical trough.—The trough of low pressure found in the vicinity of the equator between the subtropical highs of the northern and southern hemispheres. It is orientated approximately east–west and is usually found within 200 miles of the equator. The trades of both hemispheres flow without marked convergence.

Intertropical convergence zone.—The intertropical trough is called the intertropical convergence zone where the trades of the two hemispheres meet at a pronounced angle. The zone is 50 to 200 miles wide and usually 5° or more from the equator. The air on the equatorial side is trade-wind air from the opposite hemisphere. If the convergence zone is 5° or more from the equator this wind turns to westerly as it approaches the zone.

Monsoon convergence zone.—A narrow zone, usually within 200 miles of the equator, extending east–west, where the monsoon changes rapidly from an easterly to a westerly direction.

(The use of the term " trough " is not without objection. In extratropical latitudes a trough of low pressure is a term associated with bad weather and usually with activity, indeed convergence is frequently associated with a trough of low pressure, and it is generally comparatively narrow. A preferable term for the intertropical trough would be doldrums or equatorial low. The monsoon convergence zone requires evidence of reality before it is accepted.)

Transverse waves and surges

Waves in the easterlies.—These are found in regions where the easterlies are at least 15,000 to 20,000 ft. deep. They are non-frontal wave-like perturbations of this current, with trough and crest lines orientated about normal to the current, and they move with the current. In some cases, there is a shallow wedge of westerlies under the deep easterlies.

Waves in circumpolar westerlies.—The upper-level waves in the circumpolar westerlies intrude into the tropics if the westerlies themselves have extended into low latitudes. The westerlies overlap the outer parts of the trade-wind zone, with the base of the westerlies sloping up towards the equator. Waves in the upper westerlies produce eastward moving perturbations in the underlying easterlies.

Waves in equatorial westerlies.—Deep equatorial westerlies are found especially in the East Indies and the Indian Ocean, extending up to at least 15,000 or 20,000 ft., but not always based on the surface. Non-frontal wave-like perturbations occur in these westerlies and move eastwards.

Monsoon surges.—Surges in the monsoon currents (primarily in the Indian Ocean and west Pacific) are convergence zones, that travel in the direction of the monsoon current from one hemisphere into the other.

Shear lines and surges

Frontal shear lines or surge lines.—The remnants of cold fronts that have intruded into the tropics are referred to as frontal shear lines or surge lines of the trade. The density discontinuity across the old front has largely disappeared but a wind shear between the trade-wind air and the air of more recent polar origin remains. The frontal shear lines do not extend high into the atmosphere. The surge lines do not have an anticyclonic circulation above them but extend into the high atmosphere. Thus, shear lines and surge lines merely represent two different transformations of the cold front.

Other shear lines or surge lines.—Shear lines or surge lines as defined above, but whose origin cannot be traced directly to the polar front.

§24—INTERTROPICAL TROUGH AND INTERTROPICAL CONVERGENCE ZONE

The intertropical trough and intertropical convergence zone are essentially the same. By some writers they are known as the equatorial trough and equatorial front respectively. They lie along the zone of low pressure separating the subtropical high-pressure belts. In this zone, which is usually very wide and ill defined, the two trade streams flow. Cloud and precipitation will develop if either (*a*) the two air streams differ appreciably in temperature in which case a true front will form with the warmer air overrunning the colder, or (*b*) there is true horizontal convergence between the air streams.

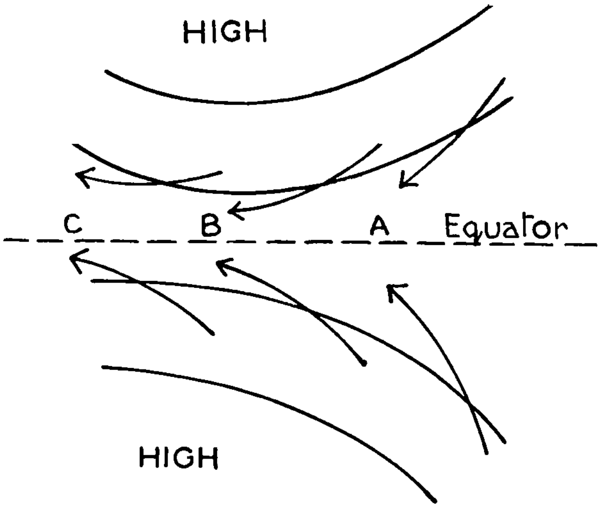

FIG. 9—INTERTROPICAL CONVERGENCE ZONE

It is often difficult to judge from the synoptic chart whether true convergence is actually taking place in this situation. Fig. 9 illustrates the apparently most favourable case, in which anticyclonic cells of the northern and southern hemispheres are situated opposite one another. The winds are indicated by the arrows. A cursory examination would lead to the conclusion that at A, near the eastern ends of the high-pressure cells, the air streams are converging, at B they are flowing side by side without convergence, and at C they are diverging. If, however, the wind between A and B is sufficiently strong, air is removed from the vicinity of A at the same rate at which it arrives and no convergence takes place at A. Similarly if near C the westward velocity is less than that at B, there may actually be convergence in the vicinity of C. Whether the air flow is convergent or divergent in any locality could only be judged from an accurate chart of stream-lines, or calculated from a close network of upper wind observations.

The equatorial belt of low pressure is often ill defined and the few isobars that appear within it have little systematic arrangement and change their pattern from chart to chart. Within this belt the air movement is often very sluggish over a wide area, only on the boundaries of which are the true trades found. Such an area is called an area of " doldrums " and on the charts it appears as a loop formed by bifurcation of the intertropical trough. Grimes[24] states that with a doldrum area the main frontal activity occurs where the trades meet the stagnant air, and he distinguishes between the cases in which the trade overruns and undercuts the doldrum air ; the former has little effect on the weather but the latter is associated with disturbed conditions.

The variation of pressure along the trough is an important factor affecting convergence. If in Fig. 9 the pressure is higher at C than at B, air moving from B to C will be retarded and convergence will take place. If pressure is relatively high to the eastward of A this effect will be increased. Walter has stated that there is reason to believe that the intermonsoonal rainfall of British East Africa tends to be heavier when pressure is relatively high in the intertropical trough over west Africa and south India.

When the axis of the trough recedes from the equator into the northern or southern hemisphere the trade wind from the opposite hemisphere crosses the equator, and subsequently, owing to the reversal of the Coriolis force, it acquires a westerly component which tends to increase with increasing latitude. The best known example is the SW. monsoon of the Indian Ocean, when the intertropical trough is displaced far to the north on account of the deep summer low which develops over north India. Other examples are in the NW. monsoons of north Australia and north Madagascar, and the SW. monsoon of west Africa. At the trough itself it would appear that there is a marked tendency for convergence between the trade of one hemisphere and the diverted trade of the other ; there is evidence to show that bad weather does occur frequently in this situation. Grimes[24] states that in the Malay region under these circumstances a sloping surface of discontinuity is formed between the opposing currents. He states that in the summer of the northern hemisphere the SW. monsoon advances, undercutting the air of the NE. trade and therefore forming a cold front. It advances northwards in a series of surges which alternate with periods of southward recession when it has the nature of a warm front. In east Africa and the Sudan the north-easterlies are usually found to overlie the south-easterlies. It would be expected that, in general, the air from the other hemisphere would, by virtue of its longer journey in low latitudes, behave as the warmer mass and therefore overrun the other, but this is not usually the case. According to Grimes the average value of the slope is about 1 in 400 for the lowest 5,000 ft., above which it is nearly horizontal. In east and west Africa and the Netherlands East Indies, however, a more usual value of the slope is 1 in 200 or 1 in 300, and it often persists up to 6 Km.

The pressure distribution is of little or no assistance in forecasting the movement of the intertropical convergence zone. The stream-line chart is the best aid, the frontal zone usually being located as the line of separation between winds having northerly and southerly components respectively. Over wide areas, however, the surface winds are often the only wind data available ; hence it is usually the surface front which is so placed, whereas observation shows that the worst weather often occurs up to 200 miles away from the surface front. It may be assumed that the front will move towards the side on which the winds are weaker, but it is difficult to estimate the speed of the motion from the observed winds. Often the front appears to jump.

For example, in the Mozambique Channel, the front may be located in the position AB, Fig. 10. A few hours later the front may be several hundred miles to the northward, say in the position CD, the winds between AB and CD having changed from northerly to southerly in the interval. Between AB and CD the full arrows represent the winds while the front is in the position AB ; the dotted arrows when it is at CD. This is not a case of ordinary frontal movement, for with the wind speeds obtaining it would be quite impossible for the front to move the large distance between AB and CD. The front at AB usually dissolves, while a new one is formed at CD, and the air which was at first flowing southwards between CD and AB now returns northwards. Similar occurrences have been noted over British East Africa. It is not at present possible to forecast them, though on one occasion at Nairobi (the occasion to which Figs. 2 to 8 refer), a few hours' warning was given by a rapid thickening of high cloud and later of medium cloud, indicating that on this occasion convergence was first manifest at high levels. This cloud sequence was a most unusual one, and it cannot be said that in east Africa rain is often preceded by anything like the prefrontal cloud sequence of temperate latitudes.

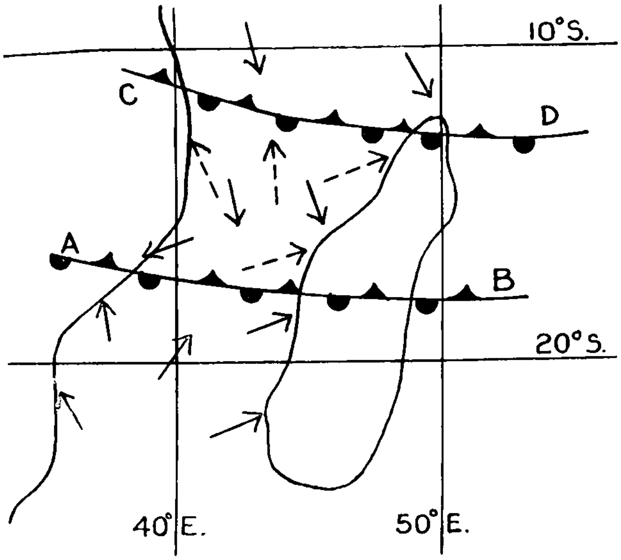

FIG. 10—DISCONTINUOUS MOVEMENT OF INTERTROPICAL CONVERGENCE ZONE

It is not clear from the definition in § 23 exactly what is meant by the monsoon convergence zone ; presumably what is intended is the zone of maximum curvature of the trajectory as air crossing the equator is diverted owing to the reversal of the Coriolis force. There does not appear to be any observational evidence for the existence of bad weather along this zone.

§ 25—TRANSVERSE WAVES AND SURGES

Transverse waves and surges comprise a number of classes of wave-like perturbations or troughs, the existence of which has only come to be recognised during the last few years. Minor perturbations of this type have long been recognised as travelling along " straight isobar " systems of temperate latitudes, and it is reasonable to expect that they will occur in the tropics also, but little is known regarding their origin.

(1) *Easterly waves* are shallow troughs travelling in a general east-to-west direction in the broad easterly currents which occur on the equatorward sides of the subtropical anticyclones. They have been described in detail for the West Indies region, principally in papers published by the University of Chicago, Institute of Tropical Meteorology.[23] The troughs are only about 1 to 2 mb. in depth and they lie approximately at right angles to the general current. The wind changes at the trough are relatively pronounced, but there is little change of temperature and humidity at the surface. Ahead of the trough there is divergence with fine weather ; behind it for a distance of up to 300 miles there is convergence, with moist air extending up to high levels

and an area of bad weather. At upper levels the trough is displaced to the eastward of the surface trough; at 20,000 ft. the average displacement is about 60 miles. There is some evidence that the pressure trough is more pronounced at 10,000 ft. than at the surface and sometimes a closed circulation is found there.

Easterly waves in the Caribbean Sea occur chiefly in summer, in the deep easterlies; they are also said to occur in both hemispheres in the western Pacific. In the western Indian Ocean minor disturbances moving from east or east-south-east have been found crossing the area from Mauritius to Madagascar and British East Africa. They occur chiefly in winter (southern hemisphere) when the SE. trade crosses the equator; it is believed that the perturbations may also cross the equator, and, turning north-eastward with the general current, may ultimately affect India. They have been detected from observations of wind changes, cloud and precipitation which were found to occur at Mauritius, eastern Madagascar, the Comoro Islands and the coast of British East Africa at intervals which suggest that they move with the general current; as a working rule it has been assumed that they move with the mean speed of the winds at 1 and 2 Km. The wind changes are shortlived but comparatively well marked, especially the backing to NE. which occurs behind the trough. The pressure changes are small and often masked by the regular diurnal variation; if, however, the pressures are corrected for the normal diurnal changes appropriate to the station and season, the troughs can be detected and followed. At Mombasa in 1944–5 considerable success was achieved by R. R. Fulwood (R.A.F.) and A. F. Coles (Naval Meteorological Service) in forecasting the movement of these troughs by graphing the departure of pressure from normal for various stations at 3-hourly intervals. It was not, however, possible to forecast changes in the intensity of the troughs, and a number which were forecast failed to appear, through filling up. The bad weather zones are usually narrow, often less than 50 miles across; in their central portions heavy cumulonimbus clouds with showers occur; on the outer boundaries there are often altostratus and stratocumulus.

The troughs appear to occur more frequently in the western Indian Ocean, where three or four may pass a station within 24 hours, than in the Caribbean Sea. On a flight from Seychelles to Mauritius in October 1944, the present writer passed through four of those zones. These had previously moved across Mauritius and had been included in the official analysis, the positions forecast being very close to those actually experienced. In east Africa these troughs were at first considered to be old polar fronts which had penetrated the subtropical high-pressure belt; this point will be further discussed in § 26. Marsden and Fairley in a paper as yet unpublished have described in detail two cases of similar disturbances affecting the Lesser Antilles in winter, and they tentatively identify them with old polar fronts which have moved round the Azores anticyclone.

The well known west African tornado may be a similar type of perturbation intensified by the frontal effect arising from the wide differences between the physical properties of the air masses involved, which are drawn respectively from the Sahara and the Gulf of Guinea, and brought into juxtaposition along the west African coast. This equatorial area is exceptional in that it is a meeting place of air streams having the widely differing characteristics necessary for the formation of a true front.

(2) *Waves in the circumpolar westerlies* occur at the surface on the poleward sides of the subtropical high-pressure belts, and at higher levels they extend to low latitudes where the westerlies overlie the trades. It is tempting to identify these with the upper parts of cold fronts associated with the depressions of middle latitudes. At Mauritius they are frequently associated with nimbostratus cloud and rain which spread from the south-west above the lower current from east-south-east. They should be indicated on the charts as upper fronts. It is possible, therefore, to have in the same part of the chart a lower front moving from an easterly, and an upper front moving from a westerly point, and crossed lower and upper fronts may occur. In the present state of knowledge, however, he would be a bold forecaster who included crossed fronts in an official analysis.

(3) *Waves in the equatorial westerlies.*—The outstanding example of a current of equatorial westerlies is the SW. monsoon of the North Indian Ocean, and the waves within it appear as the monsoon surges of India. They may perhaps be identical with the westward moving disturbances of the southern hemisphere described in (1) above, which have travelled with the general current. Elsewhere the areas covered by the equatorial westerlies are comparatively small, and there are no well known examples of perturbations occurring in them, though they possibly occur in the NW. monsoon of north Australia, originating as cold fronts in the outflow of air from the winter anticyclone of Siberia.

(4) *Monsoon surges.*—In any broad current in the tropics monsoon surges are likely to occur as a result of the strengthening of the subtropical anticyclone by the advection of a cold anticyclone from high latitudes. Apart from the weather and cloud associated with them, they could only be identified with the aid of a detailed stream-line chart. Failing this, an increase in the surface wind up-stream, especially if this appears to take place along a line across the stream, should be regarded as indicating a probable surge.

§26—POLAR FRONT IN THE TROPICS

The shear lines and surges of § 23 are mainly associated with the intrusion of the polar front into the tropics. Some consideration has been given to this question in Australia and New Zealand where the cold front, lying roughly north-south between adjacent cells of the subtropical anticyclones is known as the "meridional front". In the northern hemisphere little attention seems to have been given to the manner in which cold fronts of temperate latitudes can penetrate into the tropics, passing the barrier of the subtropical high, and in the present notes this question will be considered in some detail.

The left-hand half of Fig. 11 represents the intrusion of a cold front into low latitudes as it is usually shown on the synoptic charts. L_2 is the main depression and ABCD the associated cold front. An intense cold anticyclone H_2 spreads southwards behind the cold front, while the subtropical anticyclonic cell H_1 recedes eastwards ahead of it. The trailing portion of CD of the front is pushed far to the southward by the cold current flowing round the anticyclone H_2, and it takes up a position parallel to the stream and separates the polar from the tropical current; these flow side by side, and the difference of temperature between them is soon smoothed out, partly owing to the surface influences, but partly to subsidence in the polar current. A change of velocity only is left to mark the frontal line ; such a line is called a shear line, and the weather associated with it is usually feeble.

If a polar anticyclone spreads towards the tropics, it is to be expected that there will be a marked tendency for subsidence in the cold current associated with it, for the undercutting by the cold air of the warm air which it displaces is hindered less and less by the decreasing Coriolis deviation ; the potential energy of the cold mass is quickly dissipated in low latitudes because the small Coriolis force allows the mass to collapse owing to the more or less direct outflow near the surface. The cold current is comparatively narrow in middle latitudes and may be pictured as a powerful jet, whose energy is frittered away when it spreads out freely into the vast tropical regions.

At BC, Fig. 11, the front at the surface is almost stationary or moving very slowly south-east and its frontal characteristics have almost disappeared. Aloft, however, in this region, the westerlies of middle latitudes are still present and the upper part of the cold front B′C′ is carried on ahead of the surface front, §24. To westward of D the front is not usually shown on the charts drawn in temperate latitudes, it being assumed to have completely disappeared.

It is, however, of interest to consider, if only from a kinematical point of view, what the alignment of the front will be westward of D. The right-hand half of Fig. 11 shows the cold front PQR of the previous depression L_1 to eastward of the subtropical high H_1. TU is the warm front of the depression L_2 and kinematically it is a continuation of the front PQR. If H_1 were built up entirely in the polar air, the link between R and T would be via S′ and lie round the boundary of the polar anticyclone. The frontal alignment would then be PQRS′TU. If however, as seems more likely, H_1 built up across the front, linking up with the former subtropical high-pressure cell, the frontal linkage would be similar to RST. In this case the segment RS would move from an easterly direction. Secondary cold fronts such as QX would move in a similar way, and the westward movement of some of the perturbations discussed in § 24 might be thus accounted for.

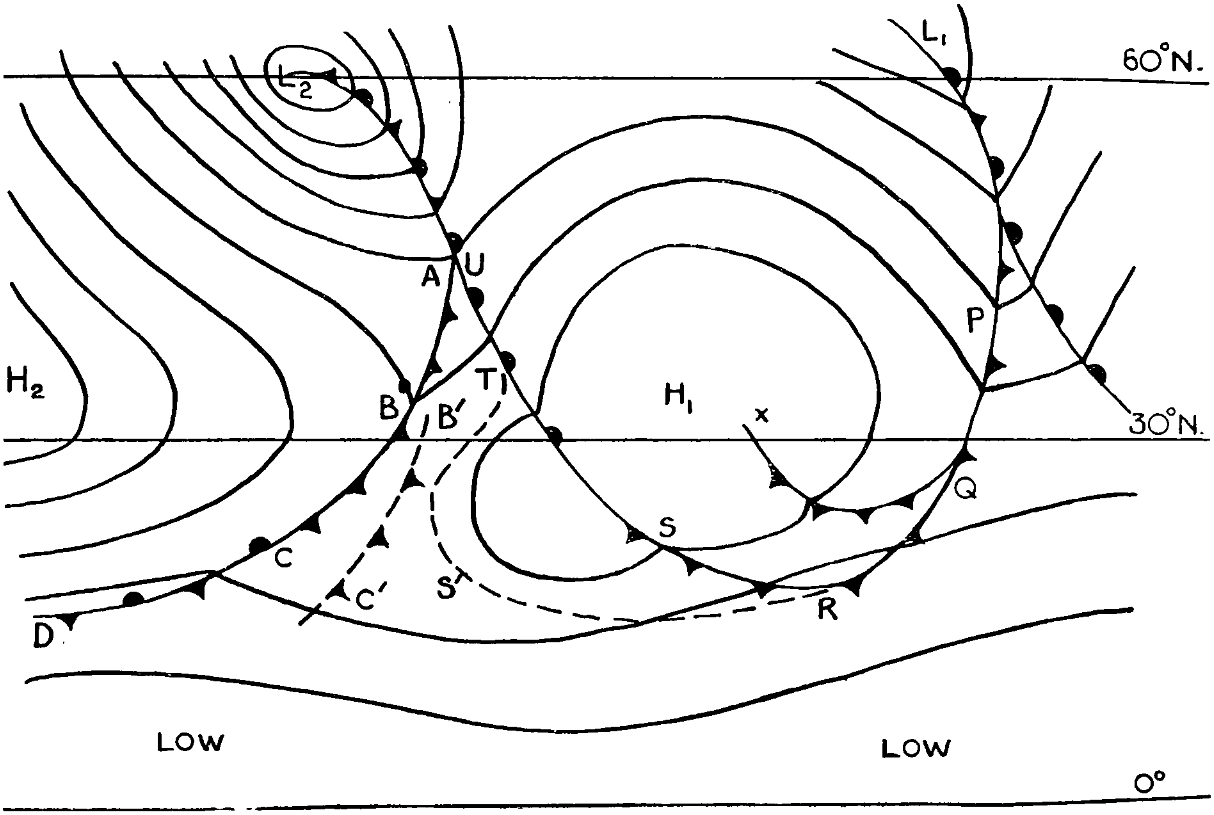

FIG. 11—PENETRATION OF THE POLAR FRONT INTO THE TROPICS

An examination of a number of weather charts of the northern hemisphere[25] confirms the possibility of the type of development described above. The published charts do not carry the analysis beyond the stage shown in the left-hand part of Fig. 11, but in a number of cases it is possible to make a reasonable extension to the analysis, obtaining configurations similar to that shown on the right of Fig. 11. The corresponding frontal alignments for the southern part of the western Indian Ocean have been discussed by the present writer.[26] The examination of charts for the northern hemisphere has shown, however, that the intrusion of cold fronts from middle latitudes into the tropics occur too infrequently to account for the large number of westward moving troughs that were observed in the western Indian Ocean, so that polar outbreaks are not the only cause of the latter, most of which ought therefore to be regarded merely as perturbations in the easterlies, concerning the origin of which little is known at present.

The preceding discussion emphasises the necessity for extending the working charts of the tropics well into temperate latitudes, for it is in these regions that the advancing cold fronts can be most easily picked up. This has been done in a series of charts of the Indian Ocean, which have been analysed in great detail in the Meteorological Office in collaboration with the Naval Meteorological Service.[2] Cold fronts from temperate latitudes are followed up to the time of fusion with the intertropical convergence zone, a typical sequence being shown in Fig. 12, which is adapted from the original publication. It is of interest to consider the movement of the cold front ABCD as shown by successive charts, particularly that of the segment ABC, which is seen to be always towards the north and north-east, and in (b) and (c) is definitely against the direction of the flow of the air. If the displacement of the front is estimated from the isobaric field, even after allowing for a component of air flow across the isobars from higher to lower pressure, it is seen that in (a) and (b) AB should move towards west or south-west and C should remain stationary, while the large north-eastward movement of BC shown between (b) and (c) appears to be quite impossible. Few upper wind observations are available for the area concerned, but over southern Madagascar they are shown in the original publication as north-easterly up to 4 Km. at least, throughout the period, and it is likely that the winds over the whole area to eastward of Madagascar would be from an easterly point up to that height also. A suggested alternative alignment of the front is shown by the dotted line B′C′ in diagrams (c) and (d) (the isobars would of course have to be altered slightly in order to fit the amended frontal position

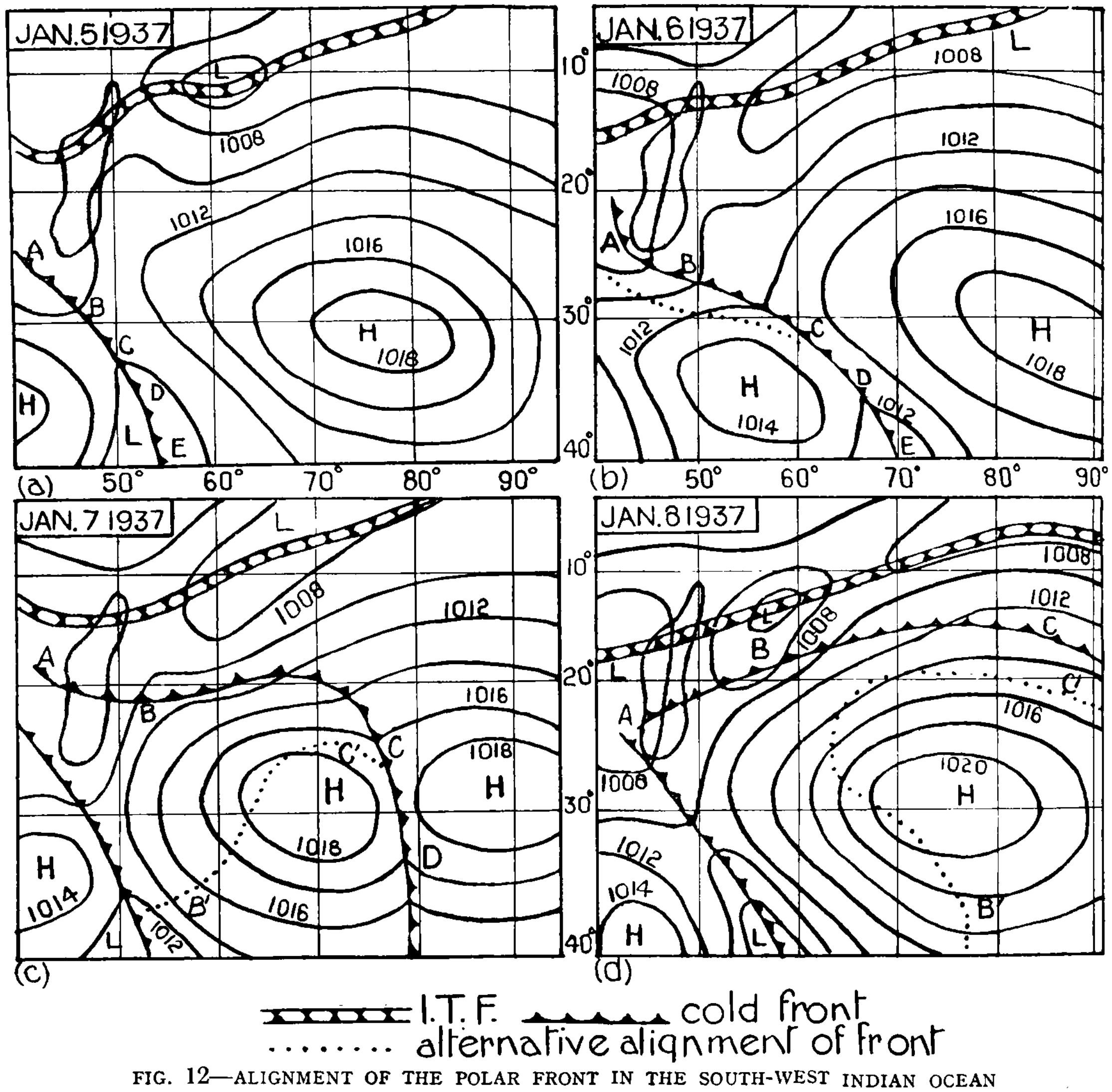
JAN.5 1937
JAN.6 1937
JAN.7 1937
JAN.8 1937
(a)
(b)
(c)
(d)
10°
20°
30°
40°
50°
60°
70°
80°
90°
1008
1012
1014
1016
1018
1020
H
L
A
B
C
D
E
B′
C′
I.T.F.
cold front
alternative alignment of front

FIG. 12—ALIGNMENT OF THE POLAR FRONT IN THE SOUTH-WEST INDIAN OCEAN

but in any case they are largely estimated). The segment of the front immediately to northward of the anticyclonic centre would be moving towards the west. The upper part of the front, just to northward of C, would, possibly, continue to move towards the east in the upper westerly current.

Only in very favourable circumstances can these fronts retain their characteristics into low latitudes, as, for example, over the Gulf of Mexico and off south China in winter ; the extensive cold land masses of North America and east Asia allow the cold air to penetrate to quite low latitudes without undergoing appreciable heating. In Australia, cold fronts moving from the south have been traced right across the continent, but except in the south they are much enfeebled and difficult to detect. Well marked cold fronts have been encountered by aircraft just to the southward of Madagascar, but they have been almost undetectable a few hundred miles farther north. It is probably true to say that the polar front has little direct significance on moving into the tropics, but it has an indirect significance, in that it may be associated with a surge of cold air which may give rise to convergence and therefore to disturbed weather.

The writer has studied in some detail a case of a surge of cold air which affected southern and eastern Africa and the south-west Indian Ocean from May 12 to 17, 1944 ; and since the effects noted are, in the main, typical of these surges, a summary of the principal features may well be given here. The surge was associated with the development of a pronounced anticyclone which had spread from high latitudes over South Africa, while the cold air spread northwards over the Transvaal, the Rhodesias and Madagascar. On May 13 a vigorous cold front, having the characteristics to a marked degree of a cold front of temperate latitudes, was encountered in about latitude 27° S. on a flight from Tuléar (south Madagascar) to Durban, and for some 400 miles behind the front the usual squally showery conditions were experienced. During the following three days the cold air spread northwards almost to the equator. But the front itself dissipated rapidly, and, though its passage was marked by a fall of upper air temperature and a shower at Tuléar, it had little or no effect on the upper air temperature in the north of Madagascar. Over Madagascar the surface temperatures were somewhat lower in the cold air than they had been at the same times on the days before its arrival, and there was an increase in shower activity well behind the front ; over the Rhodesias, while there were a few showers and thunderstorms near the front, the cold air was accompanied by much stratocumulus and a marked fall of humidity, dew points on the plateau falling below 20° F. locally, with dry-bulb temperatures about 45–50° F. Generally the indications were that over the whole area the cold air was subsiding strongly. But perhaps the most interesting feature was the surge of pressure. The pressures at a number of stations, after correction for the normal diurnal variation, were plotted against time. All showed a rise of pressure, which, as would be expected, decreased with decreasing latitude, but the surprising feature is that the rise of pressure took place almost simultaneously at all the stations during the two days May 13 to 14. Whether this is a usual feature of such surges, and if so what are its implications, is a question for further investigation.

§27—TROPICAL REVOLVING STORMS

The general characteristics, tracks, localities and times of occurrence of tropical revolving storms are well known, having been described by many writers, so that they will not be referred to further in this report. The causes of their formation, however, constitute one of the outstanding unsolved problems of meteorology. All theories are agreed that a deep moist unstable air mass is essential ; but this is a frequent condition in many tropical areas, and it would appear that in addition some form of trigger action, which is of comparatively rare occurrence, is necessary to start off a storm. No theory is adequate which does not explain the fall of pressure at the centre of the storm, and in this respect the most satisfactory explanation so far put forward is that of Durst and Sutcliffe[27], the gist of which is that the air ascends near the centre of the storm, retaining its horizontal velocity, and in the upper levels is evicted from the central regions because the geostrophic and cyclostrophic forces together outweigh the reduced pressure gradient. With regard to the synoptic aspects of the question, a number of writers, notably Grimes[20, 24] and Depperman[28] are of the opinion that the storms always develop at the equatorial convergence zone, usually at a point of bifurcation, due to the presence of a third air mass and known as a triple point. Some American writers[23] state that disturbances with closed circulations may form on most of the fronts described in §§ 24 to 27 though many of them do not develop to storm intensity. It is considered, however, that little definite is yet known of the conditions pre-disposing to storm formation, and it is only possible to say that under these conditions a risk is present. One would like to see a series of properly analysed charts showing clearly the participating fronts and air masses in a case of storm formation. Grimes[24] has put forward the further view that a cyclone only forms when there is a surge across the equator of air which in the hemisphere of origin had sufficient anticyclonic vorticity, which is conserved as cyclonic vorticity in the other hemisphere.

The main difficulty which confronts the forecaster using synoptic charts is, however, that the storms frequently form in areas devoid of observing stations, and their development and existence can only be inferred from observations at a distance. The signs of the storms are similar to those of temperate-latitude depressions—increasing wind with cyclonic circulation, increasing cloud through the sequence high, medium and low, and falling barometer ; in addition there is a pronounced swell travelling directly outwards from the centre, which can often be detected at a range of 1,000 miles. Once the existence of a storm is known the forecasting problem is not especially difficult using well known methods, and the fact that the storm is usually slow-moving enables adequate warning of the risk to be given, though it is not possible to forecast for long in advance the exact track of the central portion, which constitutes the main danger.

PART V—CONCLUSION

§ 28—SUMMARY OF PROBLEMS FOR INVESTIGATION

References are to the relevant paragraphs in this report.

(1) *Relation between wind and pressure distribution.*

(*a*) Practical investigation on lines of Gold's " Barometric Gradient and Wind Force "[3] for low latitudes ; extension of Jameson's investigation to include other seasons and other areas (crossing the equator) and to give details of the actual gradient and wind velocity calculated for each 5° square (§ 11).

(*b*) Solution of the equations of motion allowing for the variation of the Coriolis force—extension of Grimes' solution. See also classic paper by Guldberg and Mohn[29] (§ 14).

(*c*) Solution of equations for accelerated motion with constant uniform pressure field and its application to forecasting wind (§ 15 (2)).

(*d*) Solution of equations for accelerated motion by expressing the pressure variation as exponential or circular functions (Fourier series) of the time (§ 15 (3)).

(*e*) Investigation as to the reality of the oscillatory motion arising from the geostrophic acceleration. See also paper by Hesselberg on atmospheric oscillations[10] (§ 15 (3)).

(*f*) The effect of friction ; decay of atmospheric oscillations (see also paper by Hesselberg[10]) in which it is assumed that the friction is proportional to the velocity ; further investigation of the solution for unsteady motion and its extension to include initial departures from the appropriate solution for the steady state (§ 16).

(*g*) Investigation as to the effect of the movement of air across the isobars on the pressure distribution (§ 17).

(*h*) Development of technique for mapping quickly the field of convergence and divergence (§ 18).

(*i*) Applications of (*c*) and (*d*) to calculate future convergence or divergence (§ 18).

(2) *Upper air and convection.*

(*j*) Extension of temperate latitude methods of upper air analysis to the tropics (§ 19).

(*k*) Effect of high ground on the upper-level pressure distribution and its effect on wind flow (§ 19).

(*l*) Effects of convective turbulence on the upper air temperature distribution (§ 20).

(*m*) Effect of the descent of the environment on convection estimated from the tephigram (§ 20).

(*n*) Nocturnal spreading of thunderstorms from hills to plains; radiation from cloud tops, and delayed realization of latent instability (§ 20).

(*o*) Turbulent and radiative cooling of the lower layers at night; forecasting of early morning fog and low stratus (§ 20).

(*p*) Effects of advection and divergence on the upper air temperature (§ 21).

(3) *Frontal zones and discontinuities.*

(*q*) Calculation of convergence and divergence in connexion with the formation, activity and disappearance of convergence zones (§ 24).

(*r*) Detailed structure of easterly waves and their classification into convergence zones and true fronts (§ 25).

(*s*) Study of synoptic charts in connexion with the intrusion of the polar front into the tropics; study of pressure surges (§ 26).

(4) *Tropical storms.*

(*t*) Further investigation into the dynamics and thermodynamics of tropical storms and other phenomena such as line-squalls (§ 27).

(*u*) Synoptic conditions associated with tropical storms and depressions generally, by examination of particular examples (§ 27).

§29—SOME SUGGESTIONS REGARDING FORECASTING TECHNIQUE

In consideration of the points discussed in this report and the problems listed in § 28 the following suggestions are made regarding forecasting technique in the tropics.

(1) *Working charts.*—The synoptic chart should be retained; it affords the best method of representing actual weather and surface fronts. Isobars are necessary in the equatorial regions in connexion with the problems (*c*) and (*d*) of § 28, but otherwise they have little significance; in extratropical areas, to which the working charts should extend, they are the basis of the chart. Areas and zones of convergence and divergence should be indicated on the surface chart, but the details in respect of upper levels should be reserved for subsidiary charts.

(2) *Subsidiary charts.*—These should show the winds and stream-lines at standard upper-pressure surfaces. If possible isopleths of the divergence should also be drawn on these charts.

(3) *Fronts.*—It is necessary to distinguish between 3 types of discontinuity :—

(*a*) convergence zones;
(*b*) ordinary surface fronts;
(*c*) ordinary upper fronts.

These should be marked on the charts; the standard mode of representation would suffice for (*b*) and (*c*) but a new one is required for (*a*).

(4) *Pressure and tendencies.*—This question has been discussed in the Appendix to Memorandum C.E.M.46/6 of the Report of the Conference of Empire Meteorologists, London, March 4–9, 1946. It is considered that 3–hourly pressure tendencies are of value in tracing short-period disturbances such as easterly waves, but before they can be used the semi-diurnal pressure variation must be eliminated. Therefore, it is better to use differences from normal (for the time of day and month) rather than actual pressures. At high-level stations the errors involved in the

reduction of pressures to a standard level are so great that unless the stations are all close to the standard level their pressures cannot be co-ordinated ; therefore, for tracing travelling disturbances differences from normal of station-level pressure should be used.

It is of the utmost importance in view of the slight pressure gradients of the tropics and the large effect produced on the wind by a small change of pressure, that barometric readings be of the highest accuracy attainable.

(5) *Forecast charts* should be drawn where possible ; they should include fronts, and zones and areas of convergence and convection (the latter based upon the convergence, moisture and lapse rate). On many days isobars are unnecessary on these charts.

(6) *Detection of thunderstorms by sferic or radar methods.*—In the tropics cumulonimbus clouds constitute the principal hazard to aircraft in flight, and since many of the routes are over ocean or thinly populated land areas where observing posts are few, many extensive thunderstorms are missed by the reporting networks ; in addition, communication between many reporting stations and the central office is difficult and restricted to a few routine periods. The advantages of a number of suitably located sferic or radar stations for the detection of these thunderstorms are obvious.

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